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D-BRANES IN OVERCRITICAL ELECTRIC FIELDS

based on work with

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Overview

- Open strings in electromagnetic background

- Critical value of electric fields and instability

- D-brane decay

- A toy model for D-brane decay in overcritical electric fields

- S-brane point of view

- Considerations about the condensation endpoint

Open Strings in electromagnetic backgrounds [Seiberg, Witten:hep-th/9908142]

$$
\begin{aligned}
S &= \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma g_{\mu\nu}\partial_a X^\mu \partial^a X^\nu \\
&+ 2\pi\alpha'\int_\Sigma d^2\sigma \epsilon^{ab} B_{\mu\nu}\partial_a X^\mu \partial_b X^\nu
\end{aligned}
$$

Boundary conditions

$$
g_{\mu\nu}\partial_n X^\nu + 2\pi\alpha' B_{\mu\nu}\partial_t X^\mu \Big|_{\partial\Sigma} = 0
$$

Open string parameters ($2\pi\alpha' = 1$)

$$
\begin{aligned}
G^{\mu\nu} &= \left(\frac{1}{g+B}\, g\, \frac{1}{g-B}\right)^{\mu\nu} \\
\theta^{\mu\nu} &= -\left(\frac{1}{g+B}\, B\, \frac{1}{g-B}\right)^{\mu\nu} \\
G_s &= g_s\left(\frac{\det G}{\det(g+B)}\right)^{\frac{1}{2}}
\end{aligned}
$$

Purely Electric Background

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad B_{\mu\nu} = \begin{pmatrix} 0 & E \\ -E & 0 \end{pmatrix}$$

$$G_{\mu\nu} = (1 - E^2) \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\theta^{\mu\nu} = \frac{E}{1 - E^2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$G_s = g_s \left(1 - E^2\right)^{1/2}$$

CRITICAL ELECTRIC FIELD $E = 1$: the expressions above are singular.

$\longrightarrow$ (*classical*) INSTABILITY

[Nesterenko:Int.J.Mod.Phys.A4(1989) Burgess:Nucl.Phys.B294(1987) Seiberg, Susskind, Toumbas:hep-th/0005040]

(Different from *quantum* instability $\rightarrow$ pair creation [Bachas, Porrati:Phys.Lett.B296(1992)])

Neutral open strings $\to$ dipoles stretched parallel to the electric field. Large fields overcome the string tension.

Beyond the critical value virtual strings can materialize out of the vacuum and stretch to infinity

Spectrum

$$k_\mu G^{\mu\nu} k_\nu = -(n-1)$$

$$\Downarrow$$

$$-M^2 = k_\mu \eta^{\mu\nu} k_\nu = -(1-E^2)(n-1) + E^2 k_\perp^2$$

Dispersion relation

$$k_0^2 = k_1^2 + (1-E^2)k_\perp^2 + (1-E^2)(n-1)$$

Amplitudes

Physical spectrum consistently found in analytic structure of amplitudes

[Bassetto,A.T.,Valandro:hep-th/0311120]

Veneziano amplitude (four on-shell-tachyon tree amplitude) $m^2 = -2$ $(\alpha' = 1/2)$

Mandelstam variables
$s = -(k_1 + k_2)^2,\ t = -(k_1 + k_4)^2,$
$u = -(k_1 + k_3)^2$

using the metric tensor G_{ij}. $U(1)$ Chan-Paton, $s + t + u = -8$

$$\mathrm{A}(\mathrm{k}_1, ..., k_4) = \frac{\Gamma(-1-s/2)\Gamma(-1-t/2)}{\Gamma(-2-s/2-t/2)} \exp\left(\frac{-i}{2}(k_2\tilde{k}_1 + k_3\tilde{k}_1 + k_3\tilde{k}_2)\right) + \text{non-cyclic permutations}$$

where $(\tilde{k})^i = \theta^{ij}(k)_j$

COMMENT: noncommutative geometry affects the fundamental vertices of the theory.

The amplitude on the pole at $s = 0$ behaves like

$$\mathrm{A}(\mathrm{k}_1, ..., k_4) \simeq \frac{2}{s}(t-u)\,\mathrm{sin}\,\frac{k_1\tilde{k}_2}{2}\,\mathrm{sin}\,\frac{k_3\tilde{k}_4}{2},$$

If $\theta^{ij} = 0$ the residue vanishes: The two tachyons cannot couple to a photon while respecting Bose statistics

In the presence of B a tachyon-tachyon-photon vertex

$$\mathrm{V}^i(k_1, k_2) \simeq (k_1^i - k_2^i)\,\mathrm{sin}\,\frac{k_1\tilde{k}_2}{2}$$

appears, while Bose statistics and transversality

$$(k_1 + k_2) \cdot V = 0$$

are satisfied

ONE LOOP

A $=$ A(1,2,3,4) $\mathrm{tr}[\lambda_1\lambda_2]\mathrm{tr}[\lambda_3\lambda_4]+$non-trivial perm

$$A(1234)= N \int_0^1 \frac{dq}{q^3}[f(q^2)]^{-16} q^{\frac{1}{4}Kg^{-1}K} \int_0^1 d\nu_1 \int_0^{\nu_1} d\nu_2$$
$$\int_0^{\nu_2} d\nu_3\, e^{-\frac{i}{2}[k_1\theta k_2(1-2\nu_{12})-k_3\theta k_4(1-2\nu_3)]}$$
$$\left[\sin\pi\nu_{12}\prod_{n=1}^{\infty}\left(1-2q^{2n}\cos 2\pi\nu_{12}+q^{4n}\right)\right]^{-2-(s/2)}$$
$$\left[\sin\pi\nu_{3}\prod_{n=1}^{\infty}\left(1-2q^{2n}\cos 2\pi\nu_{3}+q^{4n}\right)\right]^{-2-(s/2)}$$
$$\left[\prod_{n=1}^{\infty}\left(1-2q^{2n-1}\cos 2\pi\nu_{13}+q^{4n-2}\right)\right]^{-2-(u/2)}$$
$$\left[\prod_{n=1}^{\infty}\left(1-2q^{2n-1}\cos 2\pi\nu_{1}+q^{4n-2}\right)\right]^{-2-(t/2)}$$
$$\left[\prod_{n=1}^{\infty}\left(1-2q^{2n-1}\cos 2\pi\nu_{23}+q^{4n-2}\right)\right]^{-2-(t/2)}$$
$$\left[\prod_{n=1}^{\infty}\left(1-2q^{2n-1}\cos 2\pi\nu_{2}+q^{4n-2}\right)\right]^{-2-(u/2)}.$$

$q = \exp[-\pi/\tau]$, $\nu_{rs} = \nu_r - \nu_s$ modulus and insertions on annulus

$K = k_1 + k_2$ and $f(x) = \prod_{n=1}^{\infty}(1-x^n)$

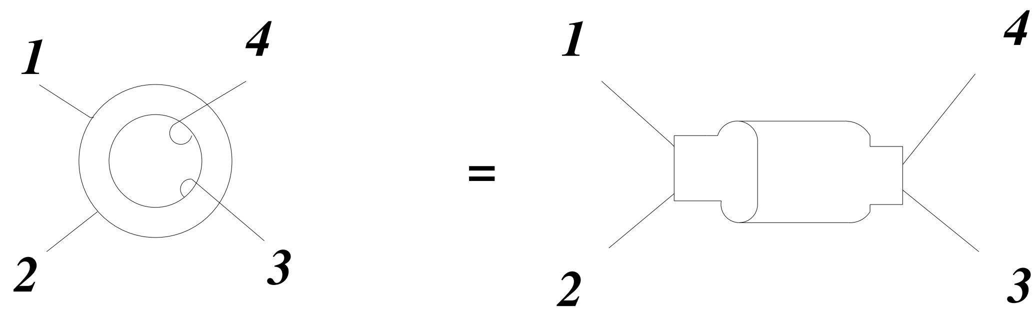


Diagram representing the amplitude $A(1,2,3,4)$

Small q region of integration

$$\text{A(1,2)} = \sum_{n=0}^{\infty} \frac{\alpha_n(s,t,u,k_1\theta k_2,k_3\theta k_4)}{(-2+n-(s_{CL}/4))}$$

$s_{CL} = -Kg^{-1}K$ squared energy variable in the closed channel

Spectrum apparently wrong: $s_{CL} = -8 + 4n = -8, -4, 0, \ldots$ but we were able to prove vanishing of residue for odd $n \rightarrow$ matching with canonical quantization

Important is symmetrization

$\alpha_1 \propto$

$$\left[\frac{4(8+t+u)\,\Gamma(-1-\frac{s}{2})\Gamma(-\frac{s}{2})\,k_1\theta k_2}{\Gamma(-\frac{s}{4}+\frac{k_3\theta k_4}{2\pi})\Gamma(-\frac{s}{4}-\frac{k_3\theta k_4}{2\pi})\Gamma(1-\frac{s}{4}+\frac{k_1\theta k_2}{2\pi})\Gamma(1-\frac{s}{4}-\frac{k_1\theta k_2}{2\pi})}\right.$$
$$-\frac{(8+t+u)\,\Gamma(1-\frac{s}{2})\Gamma(-\frac{s}{2})\,k_1\theta k_2}{\Gamma(1-\frac{s}{4}+\frac{k_3\theta k_4}{2\pi})\Gamma(1-\frac{s}{4}-\frac{k_3\theta k_4}{2\pi})\Gamma(1-\frac{s}{4}+\frac{k_1\theta k_2}{2\pi})\Gamma(1-\frac{s}{4}-\frac{k_1\theta k_2}{2\pi})}$$
$$\left.-\frac{(u-t)\,\Gamma(1-\frac{s}{2})\Gamma(-\frac{s}{2})\,k_3\theta k_4}{\Gamma(1-\frac{s}{4}+\frac{k_3\theta k_4}{2\pi})\Gamma(1-\frac{s}{4}-\frac{k_3\theta k_4}{2\pi})\Gamma(1-\frac{s}{4}+\frac{k_1\theta k_2}{2\pi})\Gamma(1-\frac{s}{4}-\frac{k_1\theta k_2}{2\pi})}\right]$$

Vanishes at $\theta = 0$ and odd under exchange $k_1 \leftrightarrow k_2$ (the Chan-Paton factors in A remain the same)

$\rightarrow$ symmetrization gets rid of unwanted singularity

- The spectrum of the bosonic open string is reversed, contains an infinite tower of tachyonic modes

- D-brane becomes unstable due to the infinite tower of tachyons

- Investigate the D-brane decay via Sen's paradigma of tachyon condensation

Boundary conformal field theory (level 2)

$$\int_{\partial\Sigma} a_{\mu\nu} \cosh(X^0\sqrt{E^2-1})\, \partial^a X^\mu \, \partial_a X^\nu \, dt$$

is a solution of linearized SFT e.o.m., provided $a_{\mu\nu}$ is symmetric, purely spatial and traceless

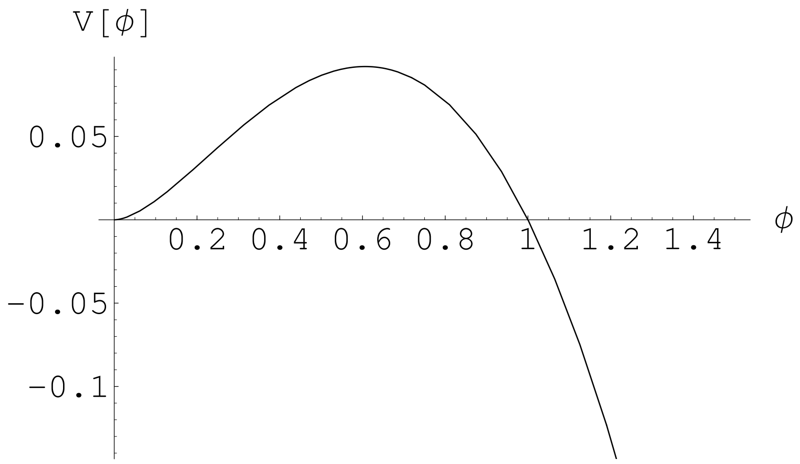
A toy model, $E = 0$ [Minahan, Zwiebach:hep-th/0008231]

Tachyon ϕ of a D25-brane

$$S = \frac{1}{g_o^2} \int d^{25}y\, dx \left[-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}(\partial_x \phi)^2 - V(\phi) \right]$$

where $y^\mu = (t, \vec{y})$, g_o open string coupling constant

$$V(\phi) = -\frac{1}{4}\phi^2 \ln \phi^2$$

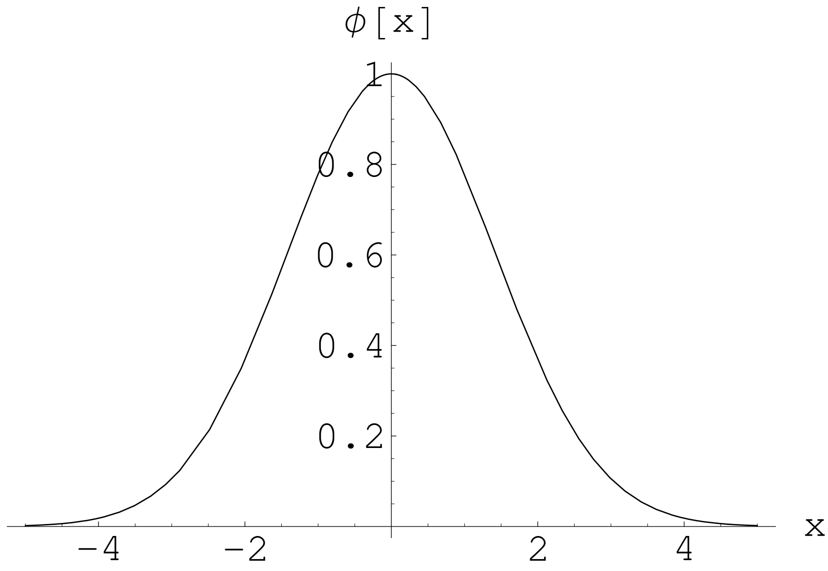


- $V(\phi)$ has a maximum at $e^{-1/2}$ (D25-brane), a local minimum at 0 (closed string vacuum)

- $V''(\phi = 0)$ diverges $\rightarrow$ decoupling of open string degrees of freedom

Gaussian lump solution independent of the y variables (D24-brane)

$$\bar{\phi}(x) = \exp\left[-\frac{x^2}{4} \right]$$



Fluctuations $\quad \phi \to \bar{\phi} + \phi$

and ansatz

$$\phi = \sum_n \xi_n(y)\psi_n(x)$$

$\Rightarrow$ Schrödinger equation for the harmonic oscillator:

$$-\frac{d^2\psi}{dx^2} + \left[-\frac{3}{2} + \frac{1}{4}x^2 \right]\psi(x) = m^2\psi(x)$$

The solutions are Hermite polynomials:

$$\psi_n(x) = \frac{1}{2^{n/2}\sqrt{n!}} H_n\left(\frac{x}{\sqrt{2}}\right) e^{-x^2/4}$$

$$m^2 = n - 1 \ , \quad n \geq 0$$

- Discrete "open string" spectrum

- Presence of a tachyon excitation $\rightarrow$ brane (lump) decay

We know that the minimum is at $\phi = 0 \rightarrow$ exact solution

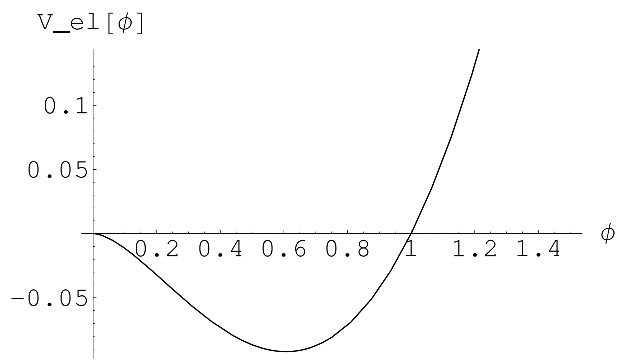
$$0 = \bar{\phi}(x) + \sum_n \xi_n \psi_n(x) \quad \Rightarrow \quad \begin{cases} \xi_0 = -1 \\ \xi_i = 0 \quad i = 1, \dots \end{cases}$$

Only the tachyon condenses

Now generalization in presence of background can be constructed $\longrightarrow$

$$S = \frac{1}{g_o^2}\sqrt{1-E^2}\int d^{25}y\,dx\Big[-\frac{1}{2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(\partial_x\phi)^2 - V(\phi)\Big]$$

- the $*$ product plays no role
- the net effect of the overcritical E field is to reverse the sign of the potential (→)



- the previous lump is still a solution of the e.o.m.
  ⇒ Schrödinger eq. not modified

Expanding around the lump in fact

$$
\begin{aligned}
S \;=\; & \frac{1}{g_o^2}\sqrt{1-E^2}\int d^{25}y\,dx\,\left[-\frac{1}{2}\Big(\frac{d\bar{\phi}}{dx}\Big)^2 - V(\bar{\phi})\right] \\
& + \frac{1}{g_o^2}\frac{1}{\sqrt{1-E^2}}\int d^{25}y\,dx\,\Big[\frac{1}{2}(\partial_0\phi)^2 - \frac{1}{2}(\partial_1\phi)^2 \\
& - \frac{1}{2}(1-E^2)\sum_a(\partial_a\phi)^2 - \frac{1}{2}(1-E^2)\phi\Big(-\partial_x^2 \\
& \qquad\qquad + V''(\bar{\phi})\Big)\phi \;\; + \ldots\Big]
\end{aligned}
$$

• D24 + fluctuations

• the squared masses are correctly multiplied by $(1-E^2)$

• Correct D-brane tension

Lump and fluctuations should sum together to the new value of the minimum

$$
\begin{aligned}
e^{-1/2} &= \bar{\phi}(x) + \sum_{n=0}^{\infty} \xi_n \psi_n(x) \\
&= \exp[-x^2/4] + \sum_{n=0}^{\infty} \xi_n \psi_n(x),
\end{aligned}
$$

Exact condensate

$$
\xi_{2n+1} = 0, \qquad \xi_{2n} = -\delta_{n,0} + \frac{2^{-n}}{n!}\sqrt{\frac{2\,(2n)!}{e}}
$$

$$
\xi_{2n} \longrightarrow \sqrt{\frac{2}{e}}(\pi n)^{-\frac{1}{4}}
$$

Unfortunately, although the ξ_n decrease for large n, they do not produce a stable minimum in level truncation

Physics of the endpoint of the condensation process?

S-brane point of view

$\partial_\tau X^0 = 0$ [Gutperle, Strominger:hep-th/0202210]

$$\mathcal{L}_{BI} \propto \sqrt{1 - E^2} \quad \overset{\text{T-duality}}{\longleftrightarrow} \quad \mathcal{L}_{part} \propto \sqrt{1 - \partial_0 X_j \partial_0 X^j}$$

S-brane: $\partial_\tau (X^1 - v X^0) = 0$, $|v| > 1$

T-duality on X^1: $\partial_\sigma X^1 - v\, \partial_\tau X^0 = 0$

To be compared with with the b.c. of a string with an electric field [Bachas:hep-th/9511043]

$$\partial_\sigma X^1 - E\, \partial_\tau X^0 = 0$$

E should be overcritical

$\Rightarrow$ we are actually considering S-branes

- Analogy with pair production of pointlike charges suggests that the endpoint corresponds to the discharge of the overcritical field [Durin,Pioline:hep-th/0507059]

- Final stage of S-branes is the flattening of their profile [Hashimoto,Ho,Wang:hep-th/0211090 Hashimoto,Ho,Nagaoka,Wang:hep-th/0303172]

  ⇒ remnants could be left: tachyon matter plus stretched fundamental strings [Mukhopadhyay, Sen:hep-th/0208142]

- Overcritical field in AdS_3 [Kluson,Nayak,Panigrahi:hep-th/0602211]

- For a full description of the phenomenon one should allow dynamics for the electric field and consider backreaction of the D-brane

- Vacuum String Field Theory [Maccaferri,Scherer Santos,Tolla:hep-th/0501011]

- Schnabl hep-th/0511286: first **analytic** solution for tachyon condensation

Conclusions

- Overcritical electric field as a well posed initial condition for tachyon condensation

- Problem of infinite tower of rolling tachyons

- Toy model $\rightarrow$ endpoint of process (true vacuum)

- S-brane picture

- Analytic solutions including all levels