M-theory, fluxes and FDAs: a no go theorem for twisted tori

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Vietri 2006

The Context

- There is a lot interest in Flux compactifications both of String Theory and M-theory
- Relation between fluxes and gauge algebras G_{gauge} in D=4 effective SUGRA.s
- There were claims that G_{gauge} could be a non trivial FDA, rather than a normal Lie algebra
- Special interest in so named twisted tori, namely in compactifications on 7-dim group manifolds

Some literature

Setting the claims

- Becker & Becker hep-th 0010282
- Hull & Reid-Edwards hep-th 0503114
- Dall'Agata Ferrara hep-th 0502066
- Dall'Agata, D'Auria Ferrara, hep-th 0503122
- D'Auria Ferrara Trigiante

hep-th 0507225

Old and New basis of the no go theorem

- Frè Class. Quant. Grav. 1(1984) L81
- Frè hep-th 0510068
- Frè & Trigiante
- Castellani, D'Auria, Frè Nucl. Phys. B239 (1984) 610
- D'Auria Frè Ann. of Phys. 157 (1984) 1

M-theory compactified on twisted tori = group-manifolds

$$\mathcal{M}_{11} = \mathcal{M}_4 \times \mathcal{G}/\Delta$$

$$e^I = e^I_J(y)\, dy^J$$

satisfy the Maurer–Cartan equations:

$$\partial e^I = \frac{1}{2} \tau^I{}_{JK}\, e^J \wedge e^K \;;\; I, J, K = 4, \ldots\; 10$$

$$V^{\hat{a}} = \begin{cases} V^r = E^r(x) & ; \; r = 0,1,2,3 \\ V^a = \Phi^a{}_J(x) \left(e^I + W^I(x)\right) & ; \; a = 4,5,6,7,8,9,10 \end{cases}$$

Kaluza Klein vectors

3-form expansion

$$\mathbf{A}^{[3]} = C^{[0]}_{IJK}(x)\, V^I \wedge V^J \wedge V^K + A^{[1]}_{IJ}(x) \wedge V^I \wedge V^J + B^{[2]}_I(x) \wedge V^I + A^{[3]}(x)$$

Expansion of $F^{[4]}$ and fluxes

$$\mathbf{F}^{[4]} \equiv F^{[4]}(x) + F^{[3]}_{I}(x) \wedge V^I + F^{[2]}_{IJ}(x) \wedge V^I \wedge V^J + F^{[1]}_{IJK}(x) \wedge V^I \wedge V^J \wedge V^K + F^{[0]}_{IJKL}(x) \wedge V^I \wedge V^J \wedge V^K \wedge V^L$$

The analysis of closure **$dF^{[4]} = 0$** relates to the theory of FDA.s, but let us consider first the solution ansatz in order to find Lorentz invariant vacua

$$\begin{aligned}
E^r &= \text{vielbein of a maximally symmetric 4-dimensional space time} \\
\Phi^I{}_J(x) &= \delta^I{}_J \\
W^I &= 0 \\
F^{[3]}_I(x) &= F^{[2]}_{IJ}(x) = F^{[1]}_{IJK}(x) = 0 \\
F^{[4]}(x) &= e\,\epsilon_{rstu}\,E^r \wedge E^s \wedge E^t \wedge E^u \quad ; \quad (e = \text{constant parameter}) \\
F^{[0]}_{IJKL}(x) &= g_{IJKL} = \text{constant tensor}
\end{aligned}$$

Three possibilities ⟹

$$\mathcal{M}_4 = \begin{cases} \mathcal{M}_4 & \text{Minkowsky space} \\ \text{dS}_4 & \text{de Sitter space} \\ \text{AdS}_4 & \text{anti de Sitter space} \end{cases}$$

Main negative result

- There are no G_7 Lie groups such that the above ansatz leads to a bona fide solution of M-theory with non vanishing flux g_{IKJL}.
- The token to prove the result is the reduction of the problem to a question of holonomy on the internal manifold M_7
- Solutions with fluxes exist **iff** M_7 has **weak** G_2 **holonomy** (= modern jargon), if it is an Englert manifold (=old jargon)
- There are no 7-dimensional Lie Groups of Englert type, *i.e.* **of weak** G_2 **holonomy**.
  - (***THEOREM in LIE ALGEBRA THEORY***)

Free Differential Algebras and Sullivan structural Theorems

FDA.s were independently discovered in Mathematics by Sullivan and in Physics by P.F. and R. D'Auria. The original name given by D'Auria and me was that of *Cartan Integrable Systems*. Later, recognizing the conceptual identity of our supersymmetric construction with the pure bosonic constructions by Sullivan, we also turned to its naming FDA. All possible FDA.s are, in a sense to be described, *cohomological extensions* of normal Lie algebras or superalgebras.

String Theory leads to p-form gauge fields that are an essential part of all supergravity multiplets in higher dimensions. The algebraic structure underlying SUGRA in D>4 is that of an FDA = Free Differential Algebra

$$d\theta^{A(p)} + \sum_{n=1}^{N} C^{A(p)}{}_{B_1(p_1)\dots B_n(p_n)}\, \theta^{B_1(p_1)} \wedge \dots \wedge \theta^{B_n(p_n)} = 0$$

$$p + 1 = \sum_{i=1}^{n} p_i$$

Sullivan's Structural Theorems

The analogue of Levi's Theorem: minimal versus contractible algebras.

Contractible Algebras

a *contractible FDA* is one where the only form appearing in the expansion of $d\theta^{A(p)}$ has degree $p+1$, namely:

$$d\theta^{A(p)} = \theta^{A(p+1)} \quad \Rightarrow \quad d\theta^{A(p+1)} = 0$$

Minimal Algebras

Denoting by $\mathcal{M}^k$ the vector space generated by all forms of degree $p \leq k$, a minimal algebra is shortly defined by the property:

$$d\mathcal{M}^k \subset \mathcal{M}^k \wedge \mathcal{M}^k$$

Sullivan's first theorem

Theorem: The most general FDA is the semidirect sum of a contractible algebra with a minimal algebra

Physical interpretation of Sullivan's first theorem

Twenty years ago I observed that the above mathematical theorem has a deep physical meaning relative to the gauging of algebras. Indeed I proposed the following identifications:

1. The *contractible generators* $\Omega^{A(p+1)} + \ldots$ of any given FDA $\mathbb{A}$ are to be physically identified with the *curvatures*
2. The Maurer Cartan equations that begin with $d\Omega^{A(p+1)}$ are *the Bianchi identities*.
3. The algebra which is gauged is the *minimal subalgebra* $\mathbb{M} \subset \mathbb{A}$.
4. The Maurer Cartan equations of the minimal subalgebra $\mathbb{M}$ are consistently obtained by those of $\mathbb{A}$ by setting all contractible generators to zero.

Differently from normal Lie algebras, FDAs already contain their own gauging.

The important point is to identify the structure of the underlying minimal algebra.

The FDA is non trivial only if the minimal algebra is non trivial.

The structure of this latter is obtained by considering the zero curvature equations, namely by modding out the contractible subalgebra

P. Fré Class. Quan. Grav. 1, (1984) L81

Sullivan's second Theorem

- Any minimal FDA $\mathbb{M}$ necessarily contains an ordinary Lie subalgebra $\mathbb{G} \subset \mathbb{M}$ whose associated 1–form generators we call e^I.
- Additional p–form generators $A^{[p]}$ of $\mathbb{M}$ are, in one–to–one correspondence with Chevalley $p+1$ cohomology classes $\Gamma^{[p+1]}(e)$ of $\mathbb{G} \subset \mathbb{M}$.
- $$\partial A^{[p]} + \Gamma^{[p+1]}(e) = 0$$

  where $A^{[p]}$ is a new object that cannot be written as a polynomial in the old objects e^I.

Chevalley cohomology

Definition of the Lie algebra

$$\partial e^I = \frac{1}{2}\tau^I{}_{JK}\, e^J \wedge e^K$$

A p-cochain is an exterior polynomial with constant coefficients

$$\Omega^{[p]} = \underbrace{\Omega_{I_1 \dots I_p}}_{\text{constant}} e^{I_1} \wedge \dots \wedge e^{I_p}$$

The algebraic action of the coboundary ∂ **:**

$$\partial\,\Omega^{[p]} = \partial\,\Omega_{I_1 \dots I_{p+1}}\, e^{I_1} \wedge \dots \wedge e^{I_{p+1}}$$

$$\partial\,\Omega_{I_1 \dots I_{p+1}} = (-)^{p-1}\,\frac{p}{2}\,\tau^R{}_{[I_1 I_2}\,\Omega_{I_1 \dots I_{p+1}]R}$$

induces a sequence of maps:

$$C^0(\mathbb{G}) \overset{\partial_0}{\Longrightarrow} C^1(\mathbb{G}) \overset{\partial_1}{\Longrightarrow} C^2(\mathbb{G}) \overset{\partial_2}{\Longrightarrow} C^3(\mathbb{G}) \overset{\partial_3}{\Longrightarrow} C^4(\mathbb{G}) \overset{\partial_4}{\Longrightarrow} \dots$$

Cohomology groups

$$H^{(p)}(\mathbb{G}) \equiv \frac{\ker \partial_p}{\operatorname{Im} \partial_{p-1}}$$

as usual...!

The 3-form and a double elliptic complex

$$\mathbf{A}^{[3]} = C^{[0]}_{IJK} V^I \wedge V^J \wedge V^K + A^{[1]}_{IJ} \wedge V^I \wedge V^J + B^{[2]}_I \wedge V^I + A^{[3]}$$

$$\mathcal{A}^{[q,p]} = A^{[q]}_{I_1 \dots I_p} \wedge e^{I_1} \wedge \dots \wedge e^{I_p}$$

D=4 space-time q-forms valued in Chevalley p-forms

$$\begin{array}{rcl} \mathbf{d} & = & d + \partial \quad ; \quad \mathbf{d}^2 = 0 \\ 0 & = & \partial\, d + d\, \partial \quad ; \quad d^2 = 0 \quad ; \quad \partial^2 = 0 \end{array}$$

Covariant derivative

$$\begin{cases} \mathcal{D}_{(W)} \equiv d - \ell_W \quad ; \quad \mathcal{D}^2_{(W)} = -\ell_G \\ G^I \equiv dW^I + \frac{1}{2} \tau^I{}_{JK} W^J \wedge W^K \end{cases}$$

Expansion of the 4 - form

$$\mathbf{F}^{[4]} = \mathbf{dA}^{[3]}$$

In a flux vacuum

$$\begin{cases} \mathbf{F}^{[4]} & = & \sqcap^{[0,4]} \\ \sqcap^{[0,4]} & = & g_{IJKL}\, e^I \wedge e^J \wedge e^K \wedge e^K \\ g_{IJKL} & = & \text{constant tensor} \end{cases}$$

$$\partial \sqcap^{[0,4]} = 0 \quad \leftrightarrow \quad \tau^L{}_{[PQ}\, g_{IJK]L} = 0$$

$$\begin{aligned} \widehat{\mathbf{F}}^{[4]} & \equiv \mathbf{F}^{[4]} - \sqcap^{[0,4]} \\ & = F^{[4]} + F_I^{[3]} \wedge V^I + F_{IJ}^{[2]} \wedge V^I \wedge V^J \\ & \quad + F_{IJK}^{[1]} \wedge V^I \wedge V^J \wedge V^K + F_{IJKL}^{[0]} \wedge V^I \wedge V^J \wedge V^K \wedge V^L \end{aligned}$$

Where →

The D=4 F-curvatures

Potentials	Curvatures
$\text{Flux}^{[0,4]} = \Pi^{[0,4]}$	$\mathcal{F}^{[0,4]} = F^{[0]}_{IJKL}\, e^I \wedge e^J \wedge e^K \wedge e^L$
$\mathcal{A}^{[0,3]} = C_{IJK}\, e^I \wedge e^J \wedge e^K$	$\mathcal{F}^{[1,3]} = F^{[1]}_{IJK}\, e^I \wedge e^J \wedge e^K$
$\mathcal{A}^{[1,2]} = A^{[1]}_{IJ}\, e^I \wedge e^J$	$\mathcal{F}^{[2,2]} = F^{[2]}_{IJ}\, e^I \wedge e^J$
$\mathcal{A}^{[2,1]} = B^{[2]}_{I}\, e^I$	$\mathcal{F}^{[3,1]} = F^{[3]}_{I}\, e^I$
$\mathcal{A}^{[3,0]} = A^{[3]}$	$\mathcal{F}^{[4,0]} = F^{[4]}$

Explicitly:

$$
\begin{aligned}
F^{[0]}_{IJKL} &= -g_{IJKL} + \tfrac{3}{2}\tau^{M}{}_{[IJ}\, C^{[0]}_{KL]M} \\
F^{[1]}_{IJK} &= \mathcal{D}_{(W)} C_{IJK} + \tau^{L}{}_{[IJ}\, A^{[1]}_{K]L} - 4\, g_{IJKL}\, W^L \\
F^{[2]}_{IJ} &= \mathcal{D}_{(W)} A^{[1]}_{IJ} + \tfrac{1}{2}\tau^{L}{}_{IJ}\, B^{[2]}_{L} - 6\, g_{IJLM}\, W^L \wedge W^M + 3\, C^{[0]}_{IJL}\, G^L \\
F^{[3]}_{I} &= \mathcal{D}_{(W)} B^{[2]}_{I} - 2\, G^J \wedge A^{[1]}_{JI} - 4\, g_{IJKL}\, W^J \wedge W^K \wedge W^L \\
F^{[4]} &= dA^{[3]} - g_{IJKL}\, W^I \wedge W^J \wedge W^K \wedge W^L + B^{[2]}_{I} \wedge G^I
\end{aligned}
$$

This provides the starting point for the analysis of zero curvature equations

The minimal FDA in D=4

The zero curvature equations $\mathcal{F}^{[p,q]} = 0$ yield a trivial FDA

$$\begin{array}{rcll} dW^I + \frac{1}{2}\tau^I{}_{JK} W^J \wedge W^K & = & 0 & ; \quad I = 1, \dots, \dim \mathbb{G} \\ d\Phi^x_{[0]} & = & 0 & ; \quad x = 1, \dots, h_3(\mathbb{G}) \\ dZ^a_{[1]} & = & 0 & ; \quad a = 1, \dots, h_2(\mathbb{G}) \\ dB^\alpha_{[2]} & = & 0 & ; \quad \alpha = 1, \dots, h_1(\mathbb{G}) \\ d\overline{A}^\ell_{[3]} & = & 0 & ; \quad \ell = 1, \dots, h_0(\mathbb{G}) = 1 \end{array}$$

In order to obtain anything non trivial as an FDA we must have a cohomologically non trivial internal flux $\Pi^{[0,4]}$

Actually by itself this does not even suffice. The cohomology class of the flux must also fulfill an additional condition to generate non trivial D=4 FDA.s

How the D=4 FDA could be non trivial

The minimal FDA $\mathbb{M}$ emerging in twisted tori compactifications of M-theory, i.e. on a group manifold $\mathcal{G}_7$ coincides with the Lie algebra $\mathbb{G}_7$ of $\mathcal{G}_7$ unless the internal flux g_{IJKL} defines a non-trivial 4-cycle of $\mathbb{G}_7$, *i.e.*

$$g_{IJKL} e^I \wedge e^J \wedge e^K \wedge e^L \equiv \Delta^{(0,4)} \in \mathrm{H}^{(4)}(\mathbb{G}_7)$$

Furthermore:

The triple contraction should be a non trivial 1–cycle.

$$\exists \, \Gamma_\alpha^{[6]} \in \mathrm{H}^{(6)}(\mathbb{G}_7) \quad \backslash \quad 0 \neq \langle i_W \circ i_W \circ i_W \, \Delta^{[0,4]} \, , \, \Gamma_\alpha^{[6]} \rangle$$

But the flux should also be a solution of M-theory field equations!! This is the PROBLEM !!!!!!!!!!!!!!!

M-theory field equations in 11=4+7

Hatted indices from 1 to 11

$$0 = \mathcal{D}_{\widehat{m}} F^{\widehat{m}\widehat{c}_1\widehat{c}_2\widehat{c}_3} + \frac{1}{96}\epsilon^{\widehat{c}_1\widehat{c}_2\widehat{c}_3\widehat{a}_1\dots\widehat{a}_8} F_{\widehat{a}_1\dots\widehat{a}_4} F_{\widehat{a}_5\dots\widehat{a}_8}$$

$$R^{\widehat{a}\widehat{m}}{}_{\widehat{b}\widehat{m}} = 6F^{\widehat{a}\widehat{c}_1\widehat{c}_2\widehat{c}_3} F_{\widehat{b}\widehat{c}_1\widehat{c}_2\widehat{c}_3} - \frac{1}{2}\delta^{\widehat{a}}_{\widehat{b}} F^{\widehat{c}_1\widehat{c}_2\widehat{c}_3\widehat{c}_4} F_{\widehat{c}_1\widehat{c}_2\widehat{c}_3\widehat{c}_4}$$

Compactification $\mathcal{M}_{11} = \mathcal{M}_7 \times \mathcal{M}_4$

Then we split the rigid index range as follows:

$$\widehat{a},\widehat{b},\widehat{c},\dots = \begin{cases} a,b,c,\dots = 4,5,6,7,8,9,10 = \mathcal{M}_7 \text{ indices} \\ r,s,t,\dots = 0,1,2,3 = \mathcal{M}_4 \text{ indices} \end{cases}$$

The general reduction of the field equations with fluxes was given in 1984 by Castellani, D'Auria and Frè *Nucl. Phys. B239 (1984) 610*
D'Auria and Frè introduced the notion of Englert spaces in 1984
in *Ann. of Physics 157 (1984) 1.*

Englert Manifolds i.e. Weak G_2 Holonomy

$$\left.\begin{aligned} R^{rs}{}_{tu} &= \lambda\,\delta^{rs}_{tu} \\ \mathcal{R}^{IK}_{JK} &= 3\,\nu\,\delta^I_J \end{aligned}\right\} \quad \longleftrightarrow \quad \left\{\begin{aligned} \tfrac{3}{2}\lambda &= -\left(24\,e^2 + \tfrac{7}{2}\,\mu\,f^2\right) \\ 3\,\nu &= 12\,e^2 + \tfrac{5}{2}\,\mu\,f^2 \end{aligned}\right.$$

Einstein eq.

$$\begin{aligned} F_{rstu} &= e\,\epsilon_{rstu} \\ g_{IJKL} &= f\,\mathcal{F}_{IJKL} \\ \mathcal{F}^{AIJK}\,\mathcal{F}_{BIJK} &= \mu\,\delta^A_B \\ \mathcal{D}^M\,\mathcal{F}_{MIJK} &= \tfrac{1}{2}\,e\,\epsilon_{IJKPQRS}\,\mathcal{F}^{PQRS} \end{aligned}$$

Englert Equation

$$\Longleftrightarrow \left\{\begin{aligned} d\Phi &= 12\,e\,\Phi^\star \\ d\Phi^\star &= 0 \end{aligned}\right.$$

7-manifolds admitting such a structure were named Englert spaces. This is equivalent to $SO(7)^+$ de Sitter holonomy in 1984 parlance to weak G_2 holonomy in modern parlance

$$\left\{\begin{aligned} \Phi^\star &\equiv \mathcal{F}_{IJKL}\,e^I \wedge e^J \wedge e^K \wedge e^L \\ \Phi &\equiv \frac{1}{24}\epsilon_{ABCIJKL}\,\mathcal{F}_{IJKL}\,e^A \wedge e^B \wedge e^C \end{aligned}\right.$$

Weak G_2 = SO(7)$^+$ de Sitter holonomy

$$\mathcal{D}_I\,\eta \;=\; m\,e\,\gamma_I\,\eta \;\Longleftrightarrow\; 0 = \nabla^{SO(8)}\eta = \nabla^{SO(7)} - m\,e\,e^I\,\gamma_I\,\eta$$

The spin connection ω^{AB} plus the vielbein e^C define on any non Ricci flat 7-manifold a connection which is actually **SO(8)** Lie algebra valued and the spinors are sections of such an SO(8) associated bundle. The stability subgroup of the $\mathbf{8_s}$ of **SO(8)** is **SO(7)$^+$**

$$\mathcal{R}^{IM}{}_{JM} = 12\,m^2\,e^2\,\delta^I_J \quad\longrightarrow\quad \nu = 12\,m^2\,e^2$$

SO(7)$^+$ de Sitter or weak G_2 holonomy implies that the manifold is Einstein

$$g_{IJKL} = \mathcal{F}_{IJKL} = \eta^T\,\gamma_{IJKL}\eta \;=\; 24\,\phi^\star_{IJKL}$$

The Flux

Three type of solutions

- The flat type
  - M_{11} = $Mink_4$ £ M_7 where M_7 = *Ricci flat*
- The Freund Rubin type
  - M_{11} = AdS_4 £ M_7 where M_7 = *Einstein manifold*
- The Englert type
  - M_{11} = AdS_4 £ M_7 where M_7 = *Einstein manifold with weak G_2 holonomy and associated flux*

Englert solution

$$\left\{\begin{array}{rcl} R^{rs}{}_{tu} & = & -30\, e^2\, \delta^{rs}_{tu} \\ \mathcal{R}^{IK}_{JK} & = & 27\, e^2\, \delta^I_J \\ F_{rstu} & = & e\, \epsilon_{rstu} \\ F_{IJKL} & = & 12\, e\, \phi^{\star}_{IJKL} \end{array}\right.$$

The Ricci tensor of metric Lie algebras

$$ds^2_{\mathcal{G}} = \eta_{IJ}\, e^I \otimes e^J \quad ; \quad \eta^{IJ} = -\,\delta^{IJ} \Leftrightarrow de^I = \frac{1}{2}\tau^I{}_{JK}\, e^J \wedge e^K$$

$$de^I - \omega^{IJ} \wedge e^K \eta_{JK} = 0$$

$$\mathcal{R}^{AB} \equiv d\omega^{AB} - \omega^{AI} \wedge \omega^{JB} \eta_{IJ} = \mathcal{R}^{AB}{}_{PQ}\, e^P \wedge e^Q$$

The Ricci Tensor

$$\begin{aligned} \mathrm{Ric}[\tau]^I{}_J \;=\; & -\tfrac{1}{4}\,\mathrm{Tr}\,[\tau_I\,\tau_J] - \tfrac{1}{4}\mathrm{Tr}\left[\tau_I\,\tau_J^T\right] + \tfrac{1}{8}\sum_{K=1}^{7}\left(\tau_K\,\tau_K^T\right)^{IJ} \\ & -\tfrac{1}{4}\sum_{K=1}^{7}\tau^I{}_{KJ}\,\mathrm{Tr}\,[\tau_K] + \tfrac{1}{4}\sum_{K=1}^{7}\tau^K{}_{IJ}\,\mathrm{Tr}\,[\tau_K] - \tfrac{1}{4}\sum_{K=1}^{7}\tau^J{}_{KI}\,\mathrm{Tr}\,[\tau_K] \end{aligned}$$

The No Go Theorem

- There is no 7 dimensional Lie algebra g_7 of weak G_2 holonomy
- Actually there are no 7 dimensional Lie algebras with a Ricci tensor with all positive and equal eigenvalues
- The proof uses Levi theorem and the form of the Ricci tensor in terms of structure constants. Two proofs
  - One by induction on the dimension of the Radical of g_7 classifies also all 7-dimensional Lie algebras
  - One uses the form of Ricci and proves that there is always at least one non positive eigenvalue.

Levi Theorem and dim g = 7 Lie algebras

Any Lie algebra $\mathbb{G}$ of dimension $\dim \mathbb{G} = n$ is the semidirect product of a semisimple Lie algebra $\mathbb{L}(\mathbb{G})$ of dimension $\dim \mathbb{L}(\mathbb{G}) = m$, called the Levi subalgebra $\mathbb{L}(\mathbb{G}) \subset \mathbb{G}$ with a solvable ideal $\mathrm{Rad}(\mathbb{G}) \subset \mathbb{G}$ of dimension $\dim \mathrm{Rad}(\mathbb{G}) = q$ so that $n = m + q$. The solvable ideal $\mathrm{Rad}(\mathbb{G})$ is named the radical of $\mathbb{G}$

For **n = 7** the possible values of q are only **q = 7, 4, 1**
because there are no semisimple Lie algebras of dimension, 7, 5, 4 or 2

Levi subalgebras

For q = 4 the Levi subalgebra is either SO(3) or SO(1,2) and we have classified all G_7 with such a Levi subalgebra

For q = 3 the Levi subalgebra is one among the following four possibilities

$$\mathbb{L}(\mathbb{G}_7) = \begin{cases} \mathrm{SO}(3) \oplus \mathrm{SO}(3) \sim \mathrm{SO}(4) \\ \mathrm{SO}(3) \oplus \mathrm{SO}(1,2) \\ \mathrm{SO}(1,3) \\ \mathrm{SO}(1,2) \oplus \mathrm{SO}(1,2) \sim \mathrm{SO}(2,2) \end{cases}$$

and the corresponding G_7 are also easily classified

Conclusions

- Twisted Tori compactifications with non trivial fluxes and hence non trivial d=4 FDA do not exist as bulk solutions of M theory.
- Could they exist in presence of sources and warp factors?
  - It is an open question under consideration.
  - This involves in any case a revisitation of Chevalley cohomology with warp factors
- What about Type IIB supergravity?
  - The analysis was never done and it should be done
  - Even the complete structure of type IIB FDA was never fully analysed and it should be.