Confining mechanisms of the QCD vacuum

Massimo D'Elia

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OUTLINE

- Confinement and the structure of the QCD vacuum
- Center vortices
- Dual superconductivity
- The order of the deconfinement phase transition as a probe for confinement

1 – The problem of confinement

Confinement of colour emerges from experimental observations as an absolute property of strongly interacting matter.

Think of the upper limit on number of observed free quarks (from Millikan-like experiments) which is 10^{-15} times smaller than what expected from the Standard Cosmological Model

Think of the evidence for a linear confi ning potential inside hadrons coming from Regge trajectories.

A full understanding of confinement starting from QCD first principles is still lacking and is one of the most challenging open theoretical problems

We have a tool to ask questions to QCD in the nonperturbative regime: numerical simulations on a lattice. We can give some criterion for confinement and verify it by numerical "experiments".

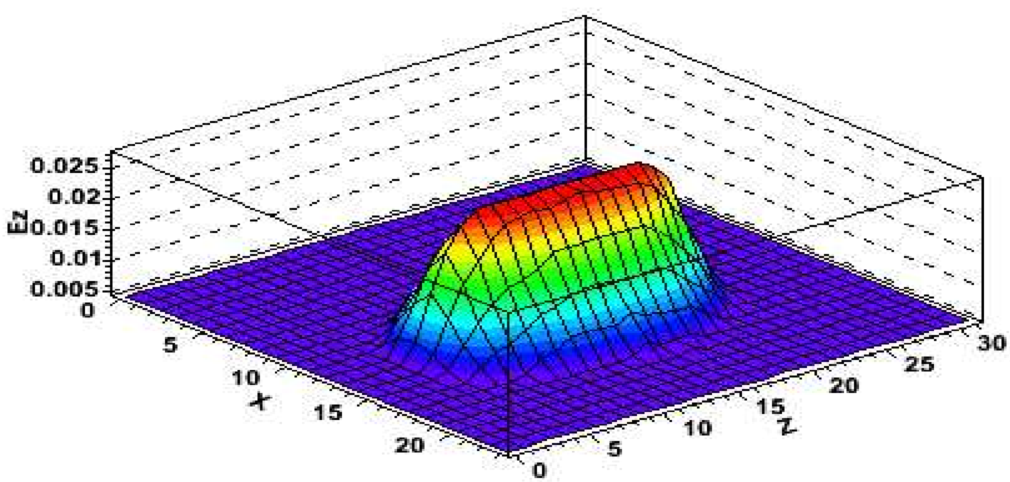
One possible criterion is the area law for the Wilson loop.

The Wilson loop $W(R,T)$ is the trace of the parallel transport over a closed path (a rectangle $R\times T$) which describes the creation and the successive annihilation after a time T of an infinitely heavy quark-antiquark pair at distance R. Its expectation value $\langle W(R,T)\rangle$ gives information, apart from UV corrections, about the static quark potential

$$\langle W(R,T)\rangle \propto e^{-V(R)T} \tag{1}$$

An area law $\langle W(R,T)\rangle \propto e^{-\sigma RT}$ means a linear confining potential.
One can also study, at finite temperature, the correlator of two Polyakov loops.

A similar approach is to look for the formation of the color flux tube in presence of two static charges:



Longitudinal chromoelectric field between two static charges, $SU(2)$ pure gauge theory

While numerical experiments confirm that QCD accounts for confinement (area law and flux tube formation are clearly observed), we still do not understand “how” it does, i.e. which is the precise mechanism leading to the phenomenon.

The area law decay can be understood in terms of strong short range vacuum fluctuations which disorder the Wilson loop.

Understanding confinement can thus be translated to understanding which are the fundamental degrees of freedom which disorder the QCD vacuum: that could eventually help in describing strongly coupled QCD in terms of weakly interacting dual variables. Several topological defects are candidate (vortices, monopoles, ...).

One approach is to directly study, on field configurations extracted from the path integral ensemble, the role played by these objects in determining confining features of the vacuum

Another approach is to try understanding confinement in terms of symmetry: can we interpret the formation of the flux tube or the condensation of some kind of topological defects in terms of the realization of some symmetry, to be tested independently?

2 – Center vortices

The center of the gauge group is believed to play a relevant role in $SU(N)$ pure gauge theories: center symmetry (corresponding to multiply all temporal parallel transport at some t_0 by a center element) is realized as a symmetry of the vacuum in the confi ned (disordered) phase and is spontaneously broken in the deconfi ned (ordered) phase: the Polyakov loop is a good order parameter and center symmetry identifi es the correct universality class for the transition in several cases (Svetitsky - Yaffe conjecture).

Center vortices are topological defects of codimension 2 (lines in 3-d, surfaces in 4-d) related to the homotopy group $\pi_1(SU(N)/Z(N)) = Z(N)$, carrying elementary magnetic fluxes belonging to the center of the gauge group. A Wilson loop linked to (pierced by) a center vortex gets a non-trvial phase factor $\in Z(N)$

A fi nite density of percolating center vortices fi lling the QCD vacuum is thus a natural way to disorder the Wilson loop. Consider for instance $SU(2)$ gauge theory. A center vortex piercing a Wilson loop contributes a (-1) to it. Supposing a fi nite density η per unit area and a random distribution of uncorrelated vortices, the probability for a Wilson loop $W(R,T)$ being pierced by n center vortices can be taken to be poissonian:

$$p(n) = \frac{(\eta RT)^n}{n!} e^{-\eta RT}$$

In a simple model in which only center vortices contribute to $\langle W(R,T)\rangle$ (as in a pure $Z(2)$ gauge model), the area law is easily obtained:

$$\langle W(R,T)\rangle = \sum_{n=0}^{\infty} p(n)(-1)^n = \sum_{n=0}^{\infty} \frac{(-\eta RT)^n}{n!} e^{-\eta RT} = e^{-2\eta RT}$$

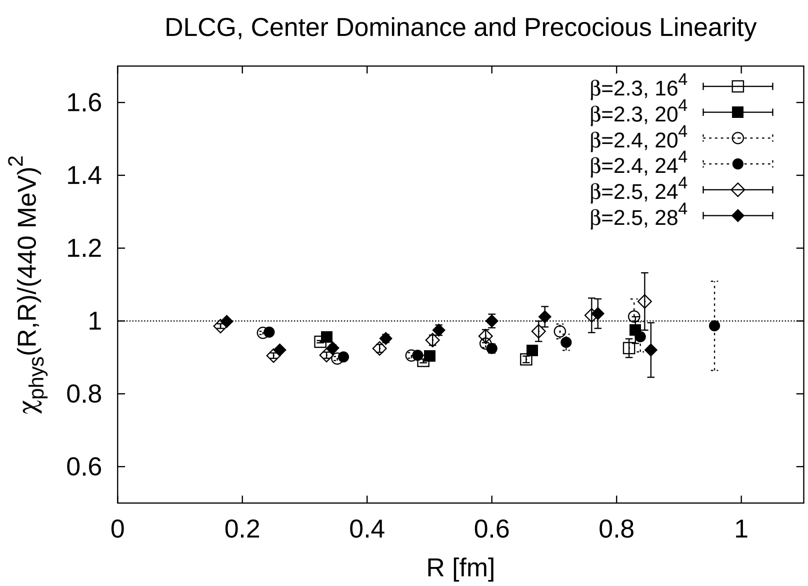
Several studies have been done on the lattice to clarify the role of center vortices. A difficulty arises in the identification of center vortices: one needs at first to project gauge variables onto the center of the gauge group. For instance in $SU(2)$ one can take $sign(\mathrm{Tr}U) \in Z(2)$.

Since this is not a gauge invariant assignement, a gauge fixing is required, with related ambiguities (Gribov copies) and arbitrariness. A widely used gauge is the maximal center gauge, in which $\sum_{\mu,x} (\mathrm{Tr}U_\mu(x))^2$ is maximized.

Center vortices are then identified by plaquettes (elementary 1×1 Wilson loops) in the center projected lattice carrying a non-trivial center element.

Modulo these problems, one can show some facts:

- The center projected theory is still confining and reproduces almost all of the string tension (center dominance)

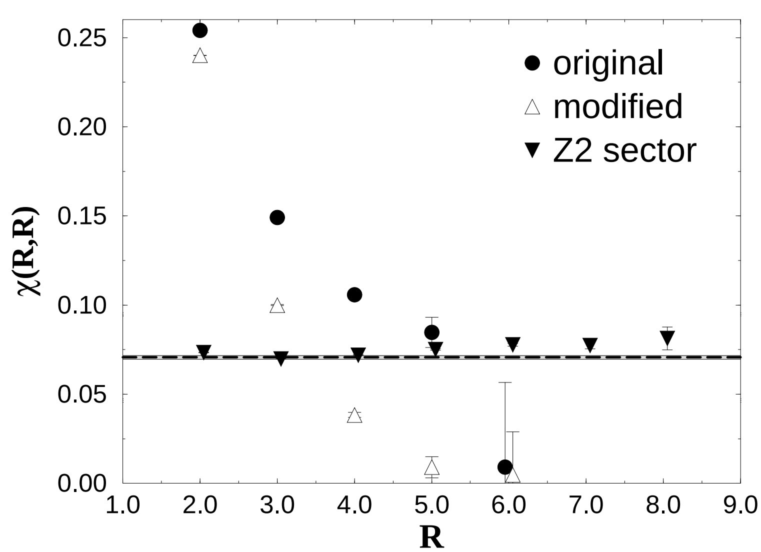


from M. Faber, J. Greensite, and Š. Olejník, JHEP 11 (2001) 053

center projected string tension compared to the full one.

- The density of center vortices scales in the continuum limit as a well defined physical quantity

- Removal of center vortices from gauge field configurations ($U \rightarrow U sign(\mathrm{Tr} U)$) leads to the absence of confinement and of other non-perturbative properties (chiral condensate, topological susceptibility, ...)



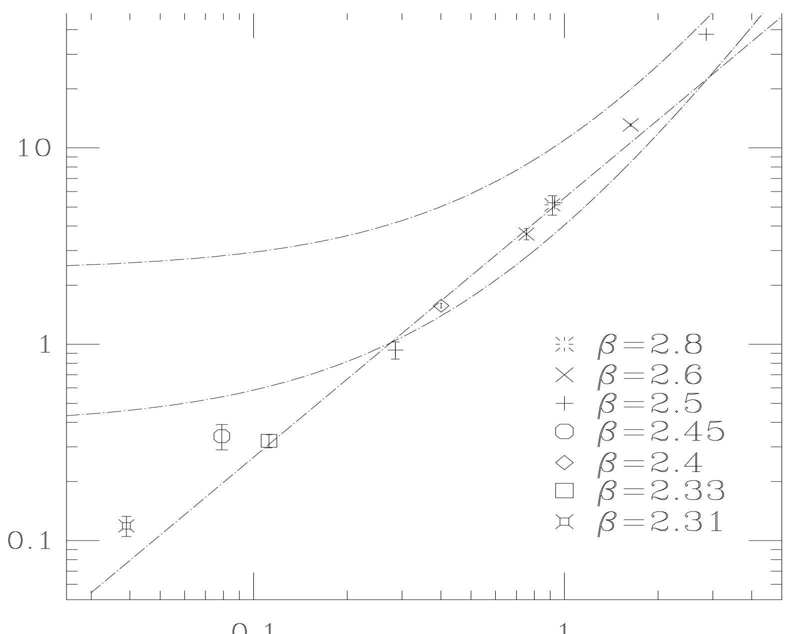
from Ph. de Forcrand and M. D'Elia, Phys. Rev. Lett. 82 (1999) 4582

Condensation of center vortices can also be studied in a gauge invariant way by means of an operator introduced by 't Hooft (the 't Hooft loop) which creates an elementary magnetic flux along a given closed contour C assigned on the dual lattice: this corresponds to adding a center phase to all plaquettes dual to some surface bounded by C.

The 't Hooft loop is in some sense dual to the usual Wilson loop, and they are exactly dual to each other in the $Z(2)$ gauge theory in $4d$.

A maximal t' Hooft loop enforces the presence of a percolating vortex sheet, its expectation value hence gives the vortex free energy, which turns to be zero in the confined phase.

The 't Hooft loop shows the correct dual behaviour (perimeter law in the confined phase and area law in the deconfined phase) leading to the determination of a dual string tension.



from Ph. de Forcrand, M. D'Elia, and M. Pepe, Phys. Rev. Lett. 86 (2001) 1438

3 – Dual superconductivity

An alternative mechanism, proposed by 't Hooft, Parisi and Mandelstam, is based on dual superconductivity of the QCD vacuum:

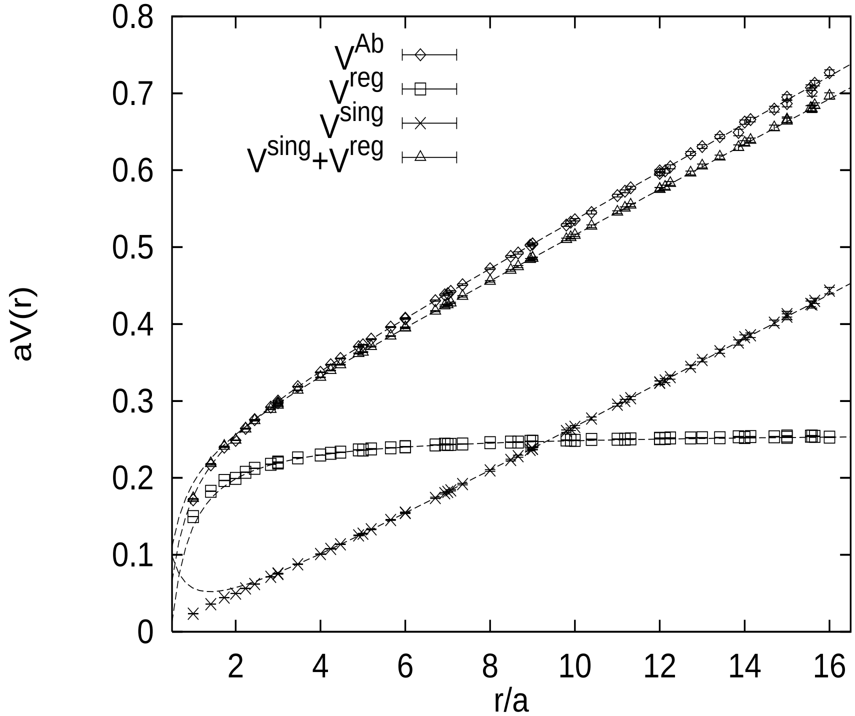
- Confinement is related to the spontaneous breaking of a magnetic symmetry produced by the condensation of some magnetically charged Higgs field (e.g. monopoles), similarly to what happens in ordinary superconductors where the electric $U(1)$ symmetry is broken by the condensation of Cooper pairs;
- The magnetic condensate filling the QCD vacuum repels electric fields out of the medium (dual Meissner effect), thus leading to the formation of flux tubes between colored charges, to the linearly rising potential, and to confinement;
- The broken magnetic group is chosen by a procedure known as *Abelian projection* ('t Hooft): a local operator $\phi(x)$ transforming in the adjoint representation is diagonalized, leaving a a residual $U(1)^{N_c-1}$ gauge symmetry. The 't Hooft tensor

$$F_{\mu\nu} \equiv \mathrm{Tr}\left\{\phi G_{\mu\nu} - \frac{i}{g}\phi\left[D_\mu\phi, D_\nu\phi\right]\right\} \qquad (2)$$

reduces to an usual abelian field tensor in the projected gauge. Gauge fields can be separated in abelian fields (transforming like photons under the residual $U(1)$ and in charged fields. Magnetic charges are associated to defects in the gauge condition or can be individuated on the lattice abelian projected configuration by measuring the magnetic flux exiting from elementary 3-cubes (De Grand - Toussaint).

The dual superconductor model has received a number of checks as well:

One attitude is to investigate whether the abelian degrees of freedom, and in particular monopoles, are those relevant for confinement by looking at *abelian* (*monopole*) *dominance* in the evaluation of observables like the heavy quark potential (string tension)



from G. Bali, hep-ph/9809351, static potentials computed in the abelian projection (V^{ab}), from monopole fi elds only (V^{sing}) and photon fi elds only (V^{reg}).

A different approach is to measure the *v.e.v.* of a magnetically charged operator $\langle\mu\rangle$ to reveal whether the magnetic symmetry is broken ($\langle\mu\rangle \neq 0$), thus confirming that dual superconductivity is at work in the confined phase and disappears at the deconfined phase transition (Pisa, Bari, Moscow);

$$\langle\mu\rangle \neq 0 \implies \text{dual superconductivity} \implies \text{confinement}$$

$$\langle\mu\rangle = 0 \implies \text{normal vacuum state}$$

The operator μ can be constructed as a translational operator

$$\mu^a(\vec{x},t) = \exp\left[i\int d\vec{y}\ \vec{E}^a_{\perp\ diag}(\vec{y},t)\vec{b}_\perp(\vec{x}-\vec{y})\right] .$$

which shifts the quantum field by the vector potential $\vec{b}_\perp(\vec{x}-\vec{y})$ produced by a Dirac $U(1)$ monopole sitting at $\vec{x}$:

$$\vec{\nabla}\cdot\vec{b}_\perp = 0\ \ ;\ \ \vec{\nabla}\wedge\vec{b}_\perp(\vec{r}) = \frac{2\pi}{g}\frac{\vec{r}}{r^3} + \text{Dirac String}\ ;$$

i.e. μ is the creation operator of a magnetic monopole in a fixed abelian projection.

The parameter $\langle\mu\rangle$ is defined on the lattice as

$$\langle\mu\rangle = \frac{\tilde{Z}}{Z}\,, \quad Z = \int (\mathcal{D}U)\, e^{-\beta S}\,, \quad \tilde{Z} = \int (\mathcal{D}U)\, e^{-\beta \tilde{S}}\,. \qquad (3)$$

$\tilde{Z}$ is obtained from Z by changing the action in the time slice x_0, $S \to \tilde{S} = S + \Delta S$. In the Abelian projected gauge the plaquettes

$$\Pi_{i0}(\vec{x}, x_0) = U_i(\vec{x}, x_0) U_0(\vec{x} + \hat{\imath}, x_0) U_i^\dagger(\vec{x}, x_0 + \hat{0}) U_0^\dagger(\vec{x}, x_0) \qquad (4)$$

are changed by substituting

$$U_i(\vec{x}, x_0) \to \tilde{U}_i(\vec{x}, x_0) \equiv U_i(\vec{x}, x_0) e^{iTb_\perp^i(\vec{x}-\vec{y})} \qquad (5)$$

The numerical determination of $\langle\mu\rangle$ is very difficult, it is expressed as the ratio of two partition functions. Instead one usually measures

$$\rho = \frac{d}{d\beta} \ln\langle\mu\rangle = \langle S\rangle_S - \langle\tilde{S}\rangle_{\tilde{S}}\,, \qquad (6)$$

the subscript meaning the action by which the average is performed. In terms of ρ

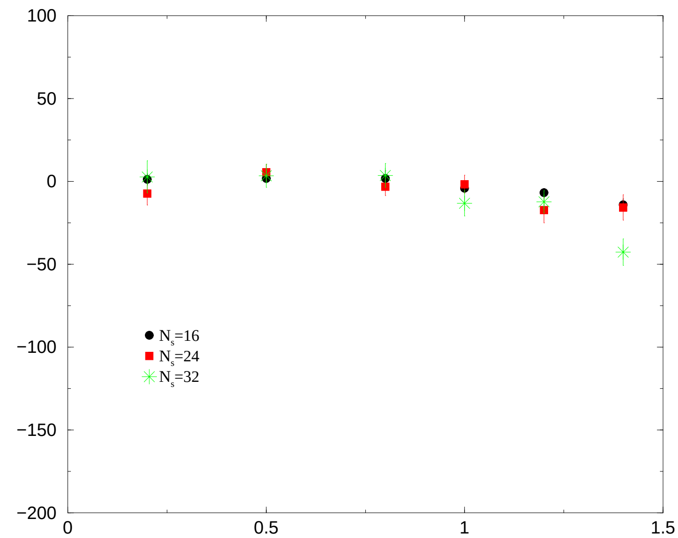
$$\langle\mu\rangle = \exp\left(\int_0^\beta \rho(\beta')\mathrm{d}\beta'\right)\,. \qquad (7)$$

$\rho \simeq 0$ in the confined phase implies $\langle\mu\rangle \neq 0$.

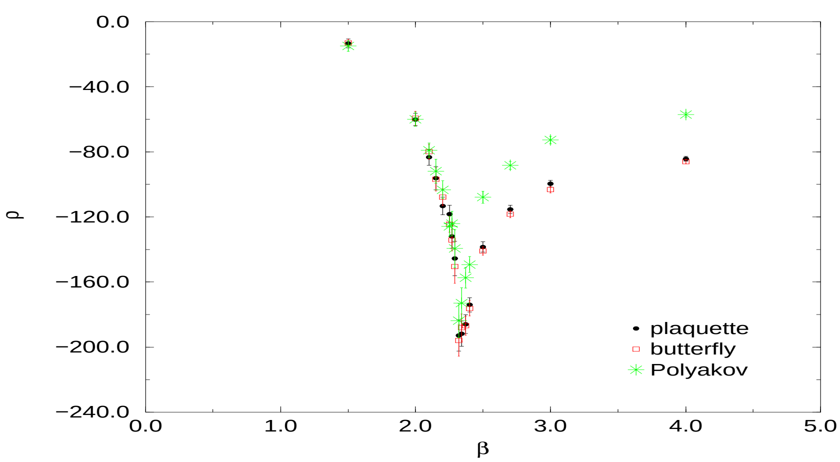
A sharp negative peak of ρ at the phase transition corresponds to a drop of $\langle\mu\rangle$

A negative value of ρ diverging in the thermodynamical limit in the deconfined phase means $\langle\mu\rangle$ being exactly zero on that side.

$\langle\mu\rangle$ has been studied in $SU(2)$ and $SU(3)$ pure gauge theory proving that $\langle\mu\rangle \neq 0$ ($\rho \simeq 0$) in the confined phase



and that $\langle\mu\rangle$ drops to zero (ρ has a large negative peak) at the deconfinement phase transition

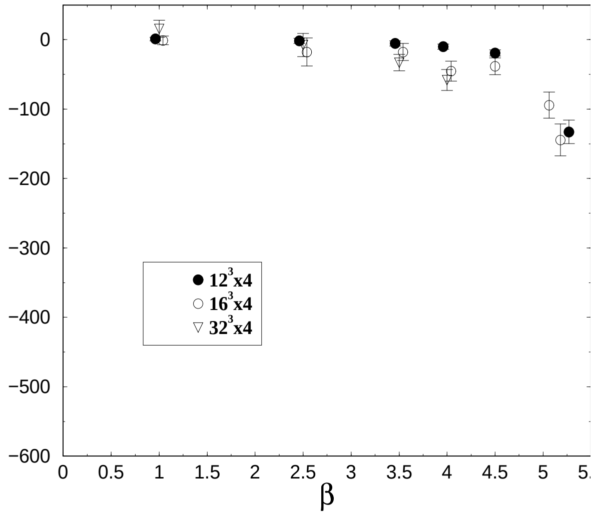


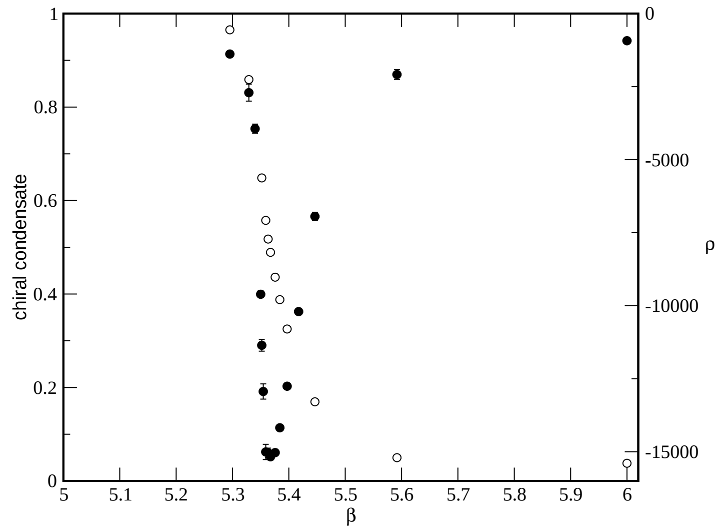
from A. Di Giacomo, B. Lucini, L. Montesi, G. Paffuti, Phys. Rev. D **61** (2000) 034503, 034504

Results are independent of the abelian projection chosen.

A finite size scaling analysis of $\langle\mu\rangle$ around the phase transition ($\langle\mu\rangle \simeq \tau^{\delta}$) gives information about the critical indexes.

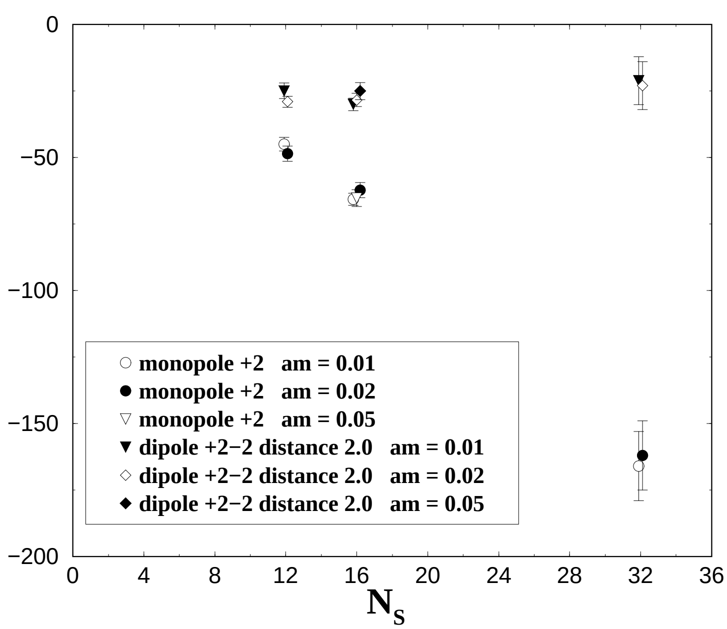
$\langle \mu \rangle$ is equally well defined in presence of matter fields. It has been shown that dual superconductivity is at work also in the confined phase of full QCD



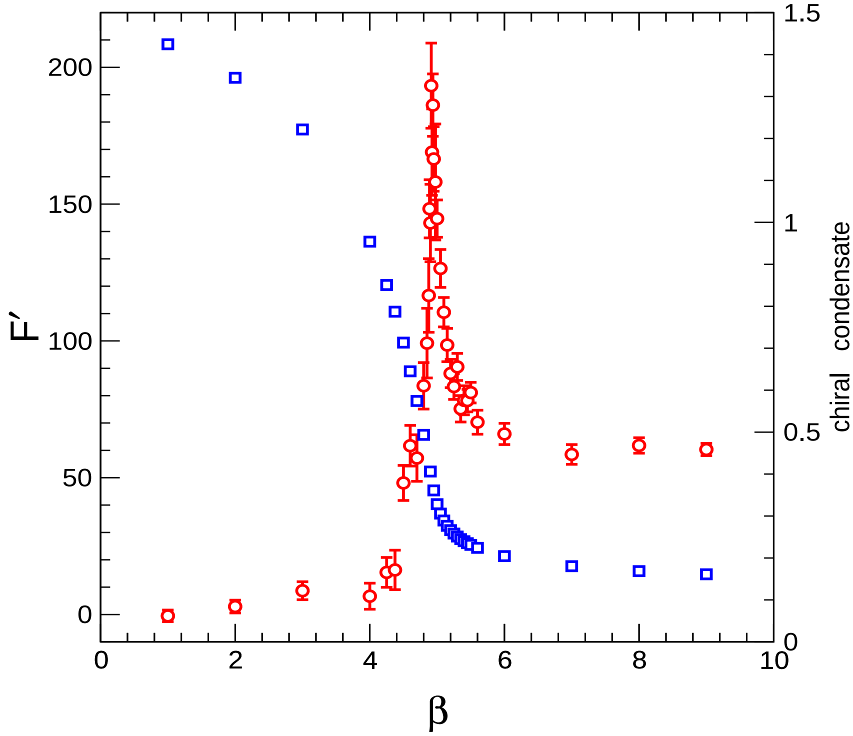


from J. M. Carmona, M. D'Elia, L. Del Debbio, A. Di Giacomo, B. Lucini and G. Paffuti, Phys. Rev. D **66** (2002) 011503

and that magnetic charge superselection is at work in the deconfined phase of the theory



Monopole condensation can be investigated also by studying the monopole effective action (free energy) in the Schroedinger functional approach, leading to similar results.



P. Cea and L. Cosmai, JHEP **0111** (2001) 064

P. Cea, L. Cosmai and M. D'Elia, JHEP **0402** (2004) 018

4 – Type I or type II dual superconductor?

Two fundamental parameters which characterize a superconductor are the correlation length ξ of the Higgs condensate and the field penetration depth λ: they determine whether the superconductor is of type I ($\xi > \lambda$) or type II ($\xi < \lambda$). In superconductors of type II there is a range of values for which an external field can penetrate in the form of Abrikosov flux tubes.

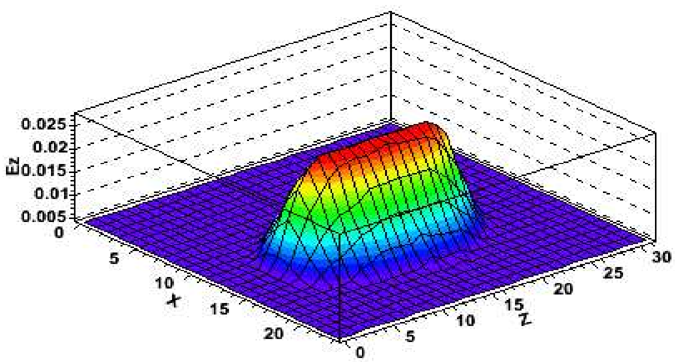
In the framework of the dual superconductor model, understanding whether the QCD vacuum behaves as a type I or a type II superconductor is a fundamental issue which can help clarifying the dynamics of color confinement and of flux tubes interactions (repulsive for type II and attractive for type I).

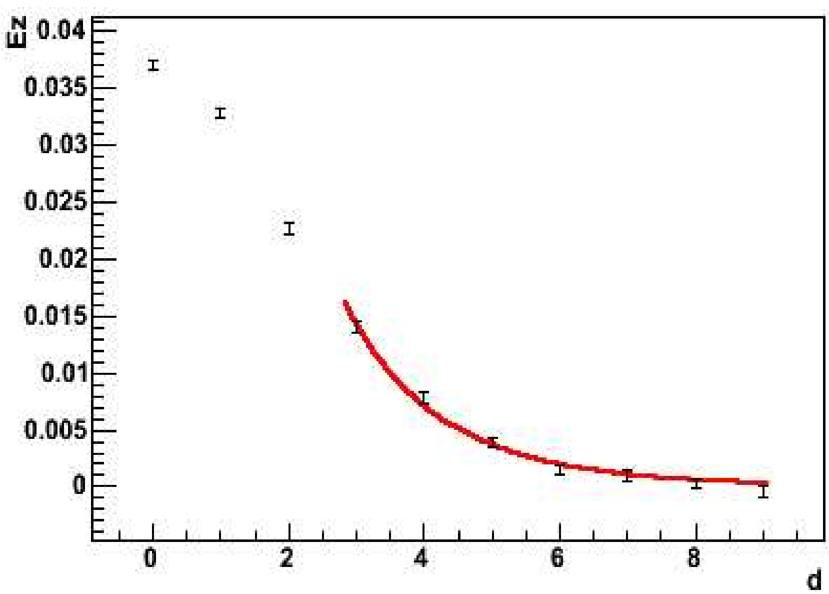
The question can in principle be answered by QCD numerical lattice simulations.

A direct way to determine the penetration depth λ is a lattice analysis of the flux tube between two static charges. The following predicition for the longitudinal field comes from the Ginzburg-Landau model in the London limit ($\lambda \gg \xi \simeq 0$)

$$E_l(x_t) = \frac{\Phi}{2\pi\lambda^2} K_0(x_t/\lambda)$$

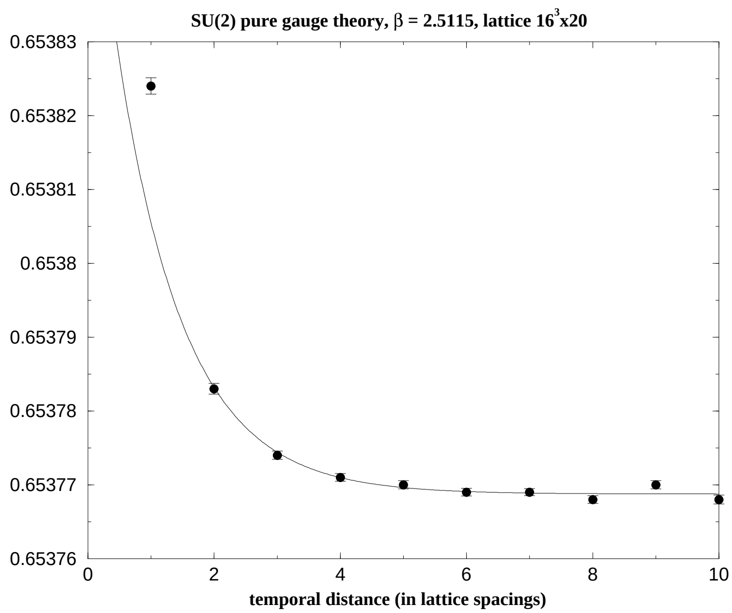
x_t is the transverse distance from the centre of the tube. In the real case (not in the London limit) this is the behaviour expected at large distances from the centre of the tube.





ξ is related to the mass of the condensing Higgs field, $m_H = 1/\xi$. A possibility is to determine m_H through the analysis of the temporal correlator of an observable directly coupled to it, like the monopole creation operator:

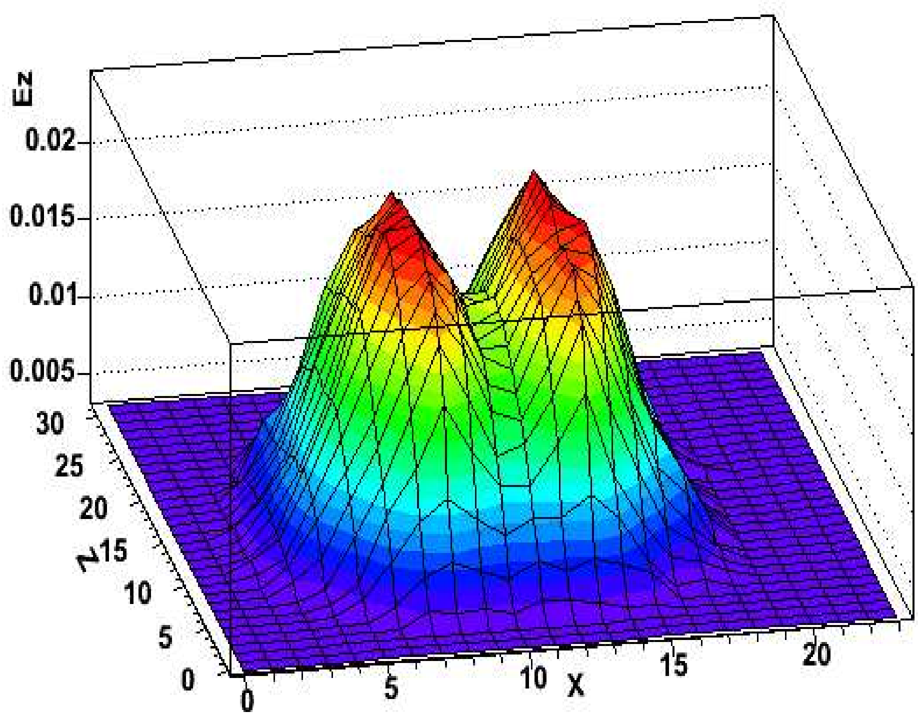
$$\langle \bar{\mu}(t)\mu(0)\rangle \simeq \langle \mu \rangle^2 + Ae^{-m_H t}$$



from A. D'Alessandro and M. D'Elia, arXiv:hep-lat/0510112

The picture emerging for pure $SU(2)$ gauge theory is that $\xi \sim \lambda$, i.e. that the QCD vacuum places roughly at the border between type I and type II.

This can be also verified directly by looking at the interaction between two parallel flux tubes, i.e. by measuring the electric field in presence of two quark-antiquark pairs with parallel axes:



No sensible deflections can be observed

5 – Comparing different mechanisms

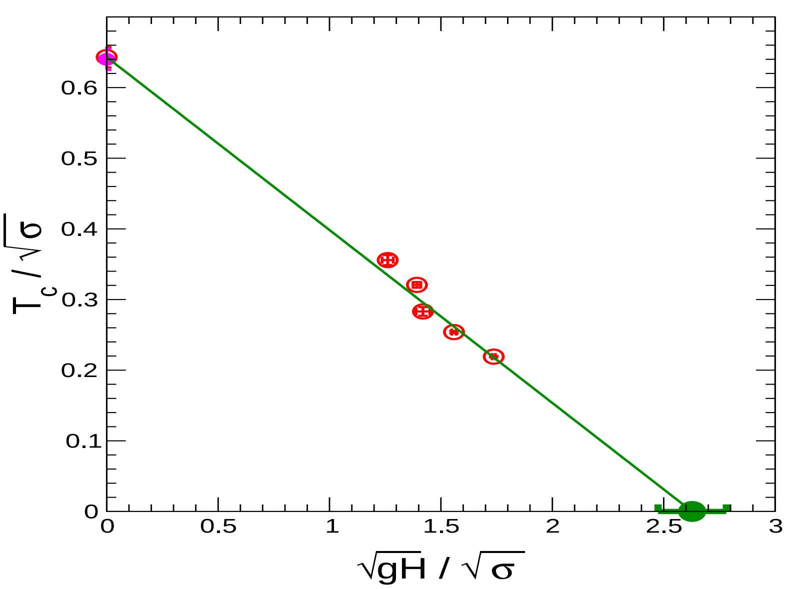
- Both center vortex condensation and dual superconductivity (monopole condensation) are two well defined mechanisms for color confinement supported by lattice data.
- It is not excluded that they are related to each other (monopole lines are located at the edges of vortex sheets). A detailed study of their interrelation has been done in simpler gauge models (F. Gliozzi and A. Rago, Phys. Rev. D **66** (2002) 074511)
- In dual superconductivity the interpretation of confinement in terms of symmetry is clearer. In the vortex model the underlying center symmetry is explicitely broken in presence of dynamical matter fields.
- Both models have some inconsistencies, like the difficulty in explaining the potential between quarks in higher representations of the gauge group.

Surely none of the two mechanisms can be considered as the definitive model for color confinement, they are more likely two different paths towards a better understanding of it.

Further studies can be clarifying, let me point out two of them:

- An interesting study is going on for theories based on gauge groups where the center is absent, like the exceptional group $G2$. (K. Holland, P. Minkowski, M. Pepe and U. J. Wiese, Nucl. Phys. B **668** (2003) 207). Center vortices are absent in such theories, but still confinement and the deconfinement phase transition are present. A study of the dual superconductivity model in this case is going on (Pisa group).

- The reaction of QCD vacuum to external fields, and in particular the variation of the deconfinement critical temperature in presence of a constant magnetic field and the existence of a critical magnetic field, is another interesting subject of investigation (P. Cea, L. Cosmai, JHEP 0508 (2005) 079).

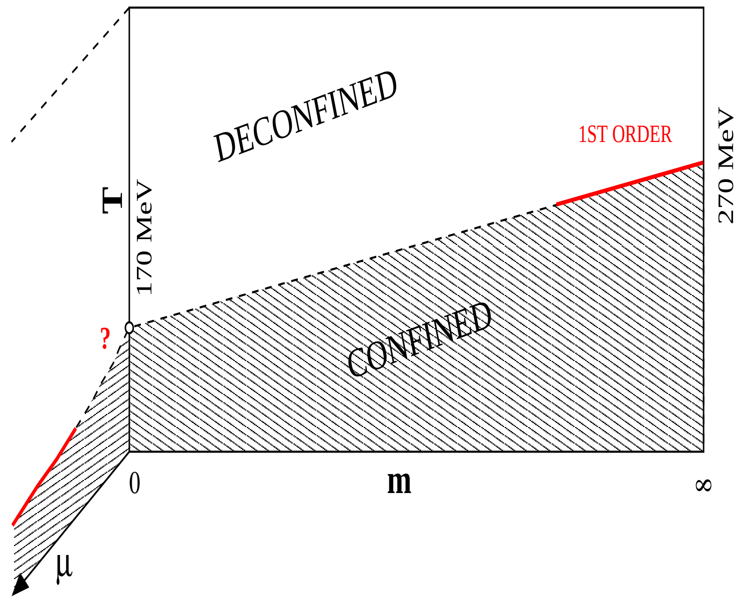


6 – Deconfinement phase transition and confinement

An interesting probe to understand the nature of confinement is the order of the deconfinement phase transition.

Experimental evidence hints at confinement as an absolute property of strongly interacting matter, which can be explained only in terms of some yet unknown symmetry of QCD. If this is the case, then deconfinement must correspond to a change of symmetry like in order-disorder transitions, and must be therefore a true phase transition.

While the situation is well established in the pure gauge theory, it must still be clarified in full QCD. The case with two degenerate light quarks, $N_f = 2$, is particularly interesting in that respect.



The phase transition is well understood at high masses ($m \geq 2.5\,GeV$) where quarks decouple: the transition is first order and the Polyakov line is a good order parameter.

At $m \simeq 0$ a chiral transition exists, where chiral symmetry is restored, $\langle\bar{\psi}\psi\rangle$ is an order parameter. At some temperature also the axial $U_A(1)$ is expected to be restored. Therefore at $m \simeq 0$ there are in principle 3 transitions (chiral, axial $U_A(1)$, deconfinement): it is not clear if they coincide. Evidence from the lattice is that they do coincide.

An effective description of the chiral transition can be given (Pisarski, Wilczek, 1983) assuming that the scalar and pseudoscalar modes are the relevant critical degrees of freedom. For $N_f \geq 3$ there is no infrared stable fixed point, the transition is expected to be first order. For $N_f = 2$ the transition is first order if the anomaly is negligible ($m_{\eta'} \approx 0$) at T_c; it can be second order with universality class $O(4)$ if the anomaly survives the chiral transition.

In the first case the transition surface around $m = 0$ is first order. In the second case instead the surface is a crossover, and a tricritical point is expected in the $\mu - T$ plane, detectable by heavy ion experiments.

The first scenario is compatible with deconfinement being an order-disorder phase transition, the second is not. In the second case there is no order parameter for confinement, and a state of a free quark can continuously be transfered below the “deconfining temperature”: confined and deconfined then lose a definite meaning.

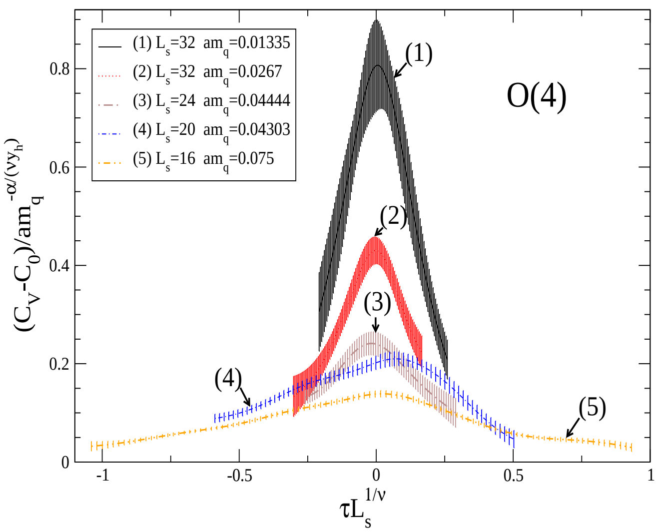
The question can be answered by a numerical study of the chiral phase transition: a careful finite size scaling (f.s.s.) analysis of thermodynamical (specific heat) quantities and of the order parameter susceptibilities is the tool to identify the critical indexes and thus the universality class.

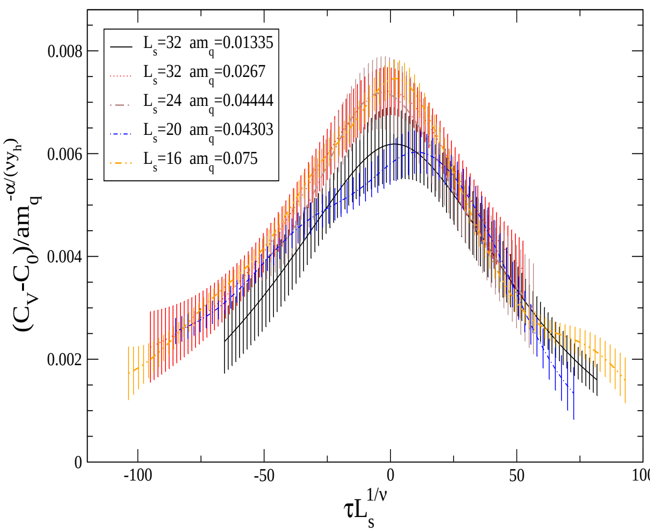
$$C_V - C_0 \simeq L_s^{\alpha/\nu} \phi_c \left(\tau L_s^{1/\nu}, am_q L_s^{y_h}\right) \ ; \qquad (8)$$

$$\chi \simeq L_s^{\gamma/\nu} \phi_\chi \left(\tau L_s^{1/\nu}, am_q L_s^{y_h}\right) \ . \qquad (9)$$

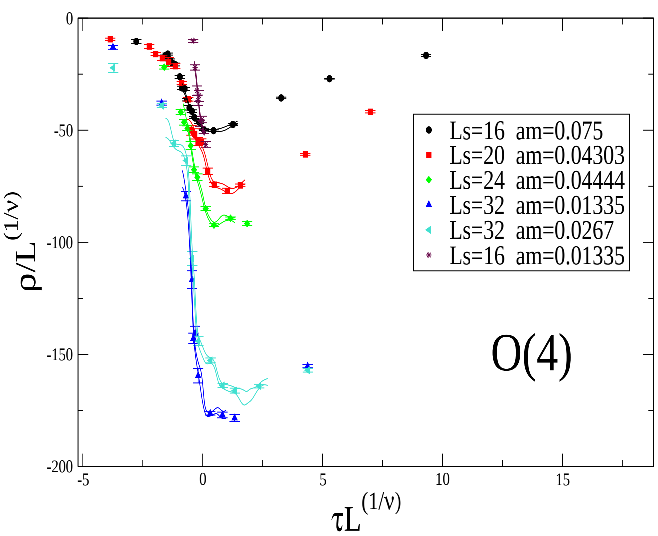
The problem is however complicated by the exceptional computer power required to study full QCD in the chiral limit and by that fact two relevant parameters enter the f.s.s. analysis: the temperature and the quark mass. For this reason the literature has been quite inconclusive, with a general tendency to prefer the $O(4)$ universality class.

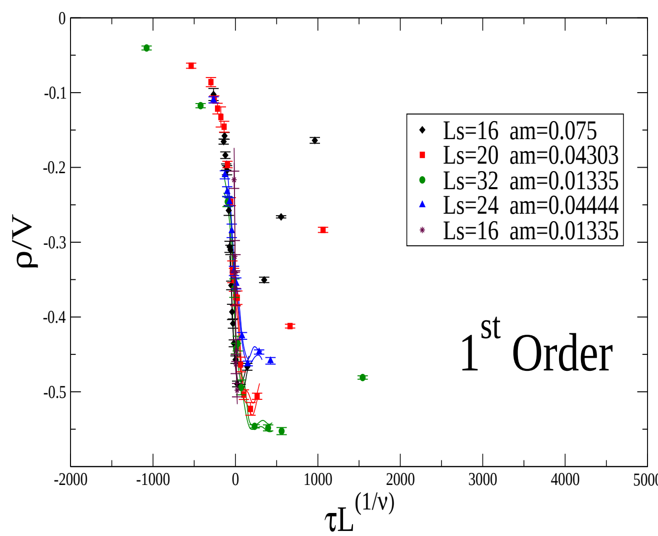
Recent studies (M. D'Elia, A. Di Giacomo and C. Pica, Phys. Rev. D **72** (2005) 114510) seem instead to exclude a second order in the $O(4)$ universality class (left) and give hints for first order



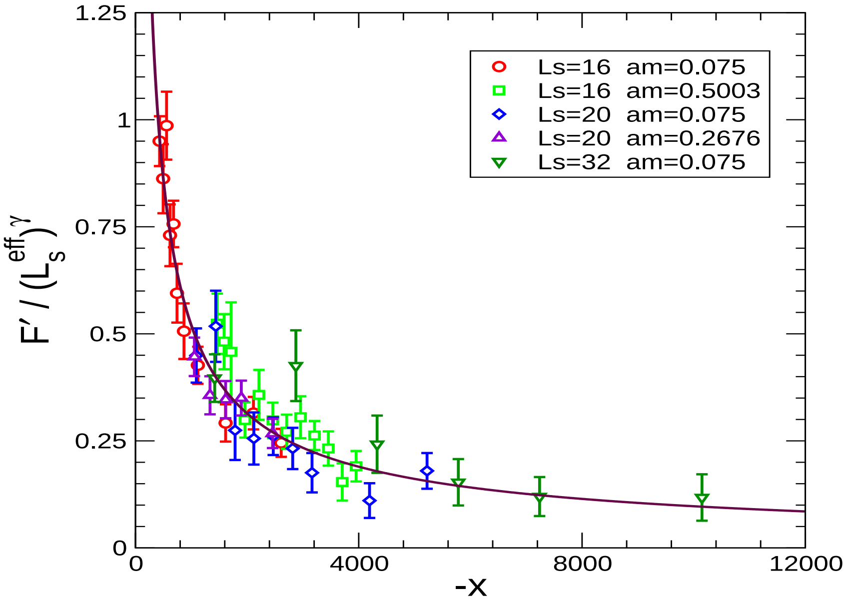


The problem can also be approached by studying some order parameter related to models for confinement. A f.s.s. analysis of the disorder parameter $\langle \mu \rangle$ related to dual superconductivity gives results consistent with a first order transition





from M. D'Elia, A. Di Giacomo, B. Lucini, G. Paffuti, C. Pica, Phys. Rev. D **71** (2005) 114502 similar results are obtained when using the monopole effective action parameter computed in the Schroedinger functional approach



from P. Cea, L. Cosmai and M. D'Elia, JHEP **0402** (2004) 018

However more extended studies with improved actions (i.e. closer to the continuum limit) are needed to fully clarify the issue.

7 – Conclusions

- Different confinement mechanisms work more or less well, but can only be considered as hints towards a complete understanding of the phenomenon

- The study of the nature of deconfinement phase transition in the full QCD theory will play an important role in elucidating what is the symmetry (if any) behind confinement of color