Recent progress in the gauge/string correspondence

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Based on works with F. Bigazzi and A.L. Cotrone

& work in progress with R. Argurio and S. Cremonesi

◇ **INTRODUCTION**

Relation between gauge theories and string theories has a long history:

1. String theory was first developed as a theory for the strong interactions, due to several string-like aspects of the latter.

2. Connection between string theory and gauge theory is also suggested by ’t Hooft large N expansion of SU(N) gauge theories.

In the mid-90's the discovery of D-branes led string theory back to gauge theories.

This opened-up a new (stringy) possibility to try and tackle gauge theory dynamics: the AdS/CFT duality. A theory of strings (in higher dimensions) emerged as to describe the strong coupling regime of a gauge theory!

The first example of such kind was of a conformal gauge theory, $\mathcal{N} = 4$ SYM, which is not confining! ... but later, it was possible to extend the duality towards confining theories, too $\longrightarrow$ today we have a string dual interpretation of QCD-like phenomena as *confinement*, *dimensional transmutation*, *chiral symmetry breaking*, and others.

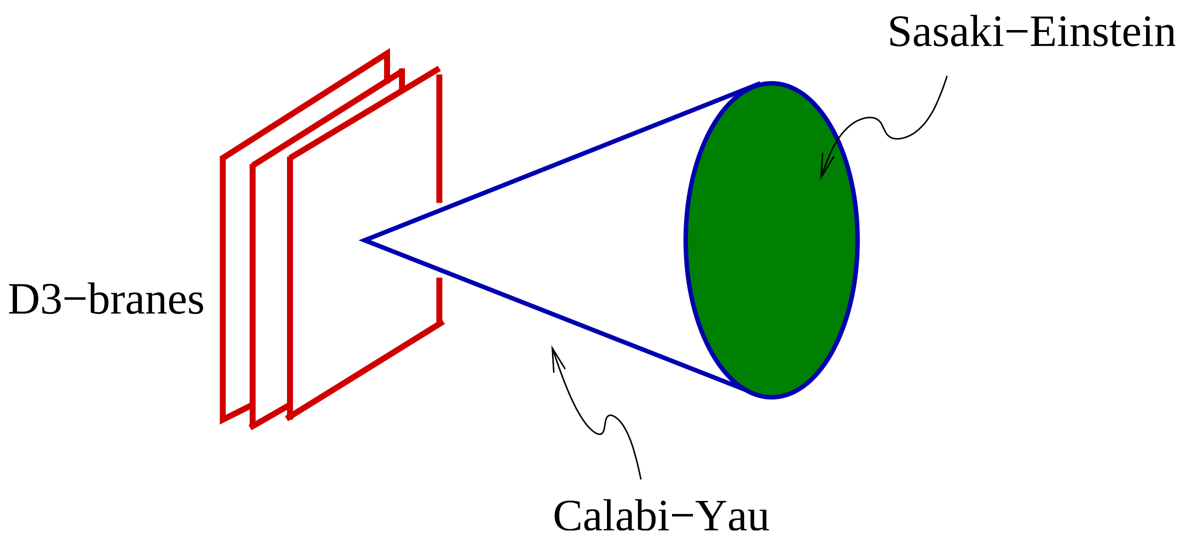
Still, we cannot yet fully realize the old dream of casting large N QCD as a string theory... but all above progress offers new hopes that an analytic approximation to QCD will be achieved along this route, and it is well worth keep trying!

The AdS/CFT correspondence is an equivalence between two (apparently unrelated) theories: a string theory in 10d and a superconformal field theory in 4d

Type IIB string theory on $AdS_5 \times X^5$ + F_5-flux	$\Longleftrightarrow$	$\mathcal{N} = 1$ 4-dimensional SCFT

X^5 is a Sasaki-Einstein manifold, i.e. the cone over X^5 is a Calabi-Yau manifold

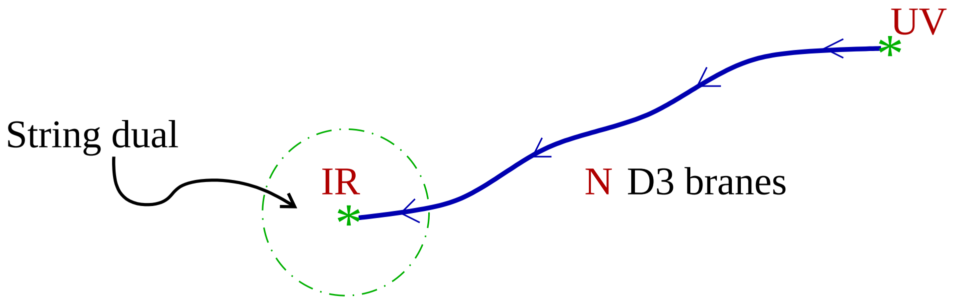
$$ds^2 = dr^2 + r^2 ds^2(X^5)$$



The only known examples, so far, were $X^5 = S^5, T^{1,1}$ and orbifolds thereof

$$S^5 \longrightarrow \mathcal{N} = 4 \text{ SYM}$$
$$T^{1,1} \sim S^2 \times S^3 \longrightarrow \mathcal{N} = 1 \text{ SCFT with matter}$$

Note: for the conifold we have an interacting IR fixed point!



Also non-conformal versions of the duality, adding fractional branes (higher dimensional branes wrapped on singular cycles): $\mathcal{N} = 2, 1$ SYM with matter.

Not only spectrum but also geometric dual of various field theory phenomena, e.g.: for $\mathcal{N} = 2$ enhanchon $\leftrightarrow$ SW monopoles locus; for $\mathcal{N} = 1$ running F_5-flux and complex deformation $\leftrightarrow$ duality cascade and confinement.

⋆ Recently new families of SE manifolds and their (smooth) metrics has been found and dual $\mathcal{N} = 1$ gauge theories have been determined

$$\begin{cases} Y^{p,q} \sim S^2 \times S^3 \\ L^{a,b,c} \sim S^2 \times S^3 \end{cases}$$

[Gauntlett et al.]

[Martelli-Sparks; Cvetic et al.]

- Correspondence between CY and CFT
- New checks for AdS/CFT
- New models for non-conformal theories with a geometric dual

Today I focus on the first class.

What's new w.r.t. the conifold?

- AdS/CFT: (super)symmetries and ABJ-anomaly vanishing conditions are not sufficient to characterize the IR fixed point $\longrightarrow$ independent checks for QFT techniques, as a-maximization (and a description of its geometric dual, *i.e.* Z-minimization). [cfr. D. Martelli's talk]

- non-AdS/non-CFT: similarly to the conifold there is a cascade of Seiberg dualities but the far IR is very different $\longrightarrow$ no susy vacua at finite field vev's, either dynamical supersymmetry breaking or runaway behavior at the bottom of the cascade.

◇ **THE CONFORMAL CASE**

The $Y^{p,q}$ ($p > q$ integer co-prime) are an infinite family of Sasaki-Einstein manifolds with topology $S^2 \times S^3$ and smooth metrics. The corresponding CY cones are *toric*.

The metrics have isometries $SU(2) \times U(1) \times U(1)$. The full sugra background has an extra abelian symmetry due to $F_5 = dC_4$: the KK reduction on S^3 provides an extra abelian symmetry.

The volume is a (q, p-dependent) rational/irrational fraction of the volume of a unit S^5, $V(Y^{p,q}) = f(p,q)\pi^3$ (this depend on $\sqrt{4p^2 - 3q^2}$).

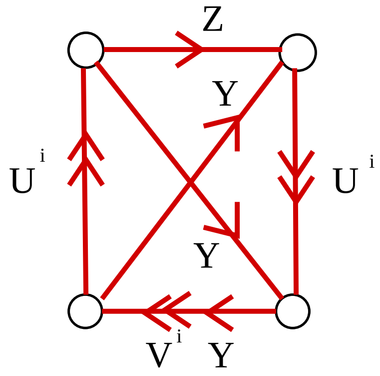
There are four supersymmetric 3-cycles, Σ_A, which are U(1)-bundles on suitable S^2 's. The volumes are (q, p-dependent) fractions of the volume of a unit S^3, $V(\Sigma_A) = g_A(p,q)\pi^2$.

There are (singular) limits which are useful to confront with others dual pairs: $Y^{p,0}$ is $T^{1,1}/\mathbf{Z}_p$ while $Y^{p,p}$ is $S^5/\mathbf{Z}_{2p}$.

The gauge theories dual to $Y^{p,q}$ are chiral (but non-anomalous) $\mathcal{N} = 1$ SCFT's with gauge group $SU(N)^{2p}$, $4p + 2q$ chiral superfields in the bi-fundamental rep. and a superpotential which is a sum of $2p + 2q$ cubic and quartic terms.

[Benvenuti et al.]

These theories can be represented in terms of quiver diagrams

$Y^{2,1} =$ 

Important to us: the global non-R symmetry group includes an anomaly-free $U(1)^2$ factor. In these cases symmetries and ABJ-anomaly vanishing conditions do not fix the R-charge univocally since abelian factors mix with the R-symmetry $\longrightarrow$ a-maximization needed to fix the correct R-charge at the IR fixed point.

There are some basic AdS/CFT relations which one expects to hold. In particular:

i) the central charge is related to the volume of X^5, $a = c \sim 1/V(X^5)$

ii) D3-branes *wrapped* on susy 3-cycles correspond to *baryons* and the volume of 3-cycles is related to the baryons R-charge, $\Delta(B) = \frac{3}{2}R(B) \sim V(\Sigma)$

iii) the *symmetries* of the supergravity background correspond to the *global symmetries* of the dual SCFT.

⋆ Full matching between gauge theory and geometric expectations: central charges, baryon charges (and dimensions), global symmetries are exactly predicted by the sugra dual!

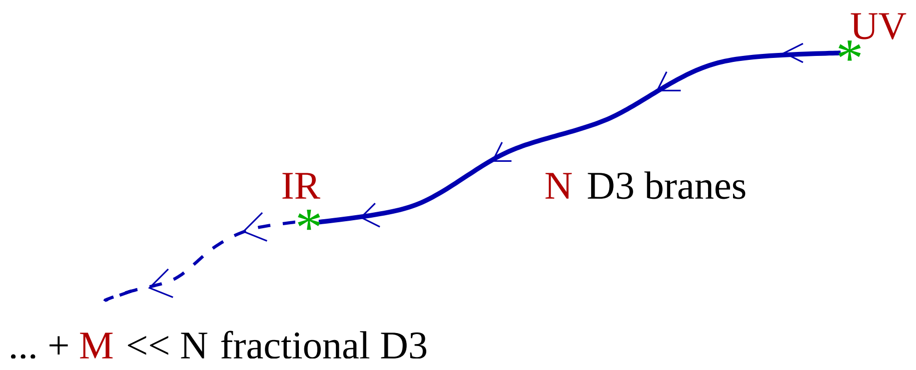
[Martelli-Sparks, Bertolini-Bigazzi-Cotrone, Benvenuti et al.]

Note: this is quite non-trivial for these gauge theories. For this reason these matchings have also been seen as a non-trivial test for the validity of the (field theory) a-maximization conjecture.

◇ **BREAKING CONFORMAL INVARIANCE**

The topology of the base space $\sim S^2 \times S^3$ allows to introduce fractional branes, i.e. higher dimensional branes wrapped on $S^2 \rightarrow$ add $M(<< N)$ fractional D3 branes: gauge theory and sugra dual change.

Conformal invariance is broken and the theory flows away the IR fixed point



Note: at leading order in M/N anomalous dimensions of fundamental fields are the ones for $M = 0$, i.e. those computed in the conformal case (γ gets corrections of order at least $\mathcal{O}(M/N)^2$).

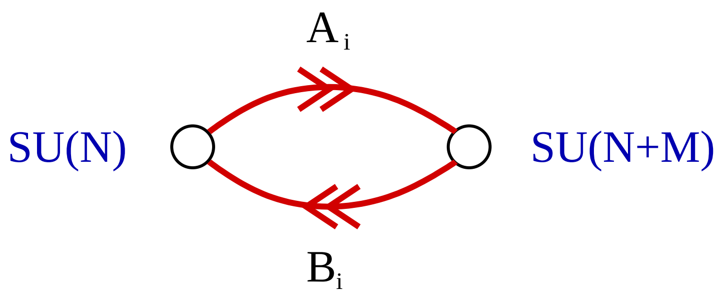
Fractional branes drive the IR dynamics. In $\mathcal{N} = 1$ dualities there exist two kinds of fractional branes, according to the IR behavior they produce:

1. deformation fractional branes: induce a complex deformation and give rise to susy (confining) vacua; no ADS superpotential.

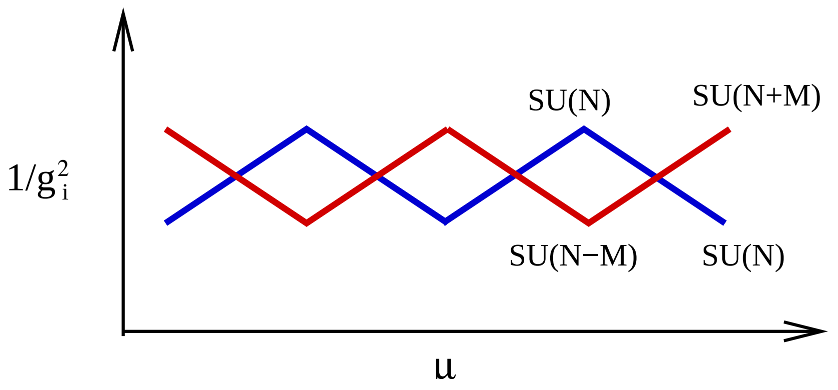
2. supersymmetry breaking (SB) fractional branes: no complex deformation, no susy vacua; give rise to ADS superpotential and may induce either *dynamical supersymmetry breaking* or *runaway behaviour*.

The conifold story: duality cascade and confinement (*without a mass gap*)

The gauge theory has group $SU(N+M) \times SU(N)$, matter in the bi-fundamental rep. and a quartic superpotential $W = \epsilon^{ij}\epsilon^{kl}\mathrm{Tr}\,[A_i B_k A_j B_l]$

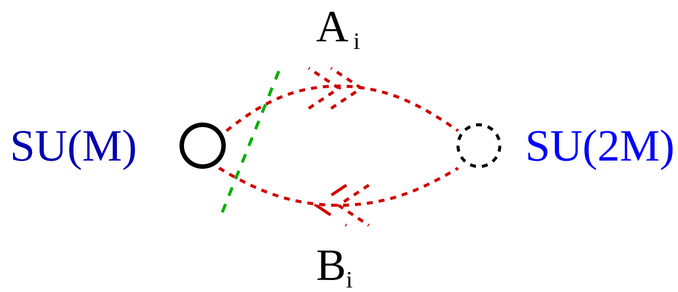


The $SU(N+M)$ is UV-free and has $\beta_{\frac{1}{g^2}} = 3M > 0$ while $SU(N)$ is IR-free and has $\beta_{\frac{1}{g^2}} = -3M < 0$. The theory is *self-similar* and undergoes a duality cascade!

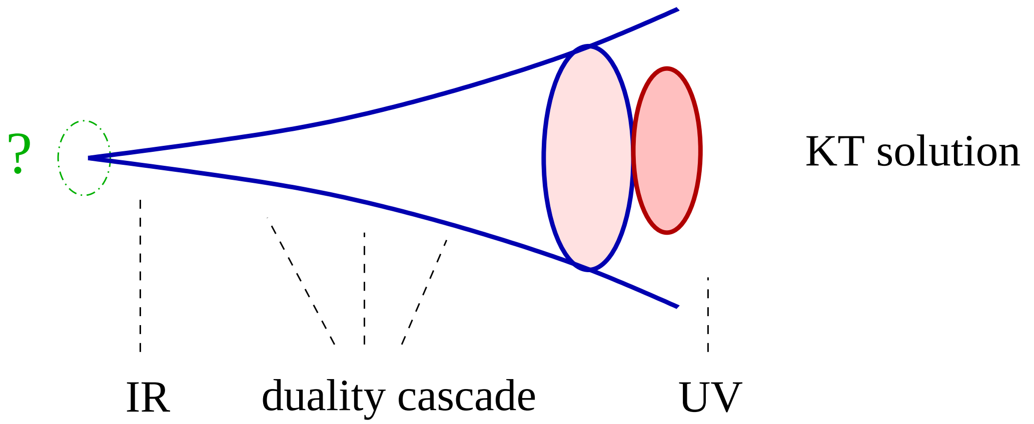


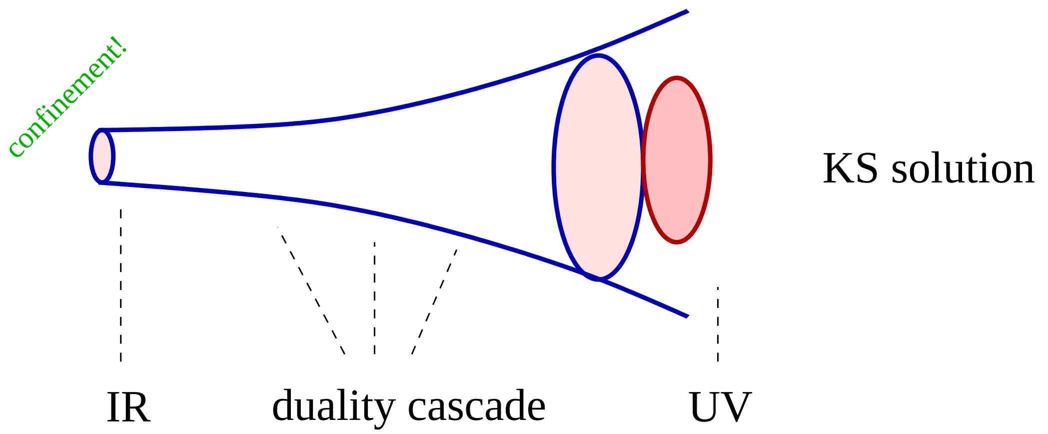
This goes through until N becomes order M. For $N = M$ the bigger gauge group has $N_f = N_c$, the quantum moduli space is modified and the theory is *not* self-similar anymore: the cascade stops.

The theory $SU(2M) \times SU(M)$ admits solutions of the F-eqs. along the baryonic branch where $U(1)_B$ gets broken and chiral fields get a mass: in the IR theory reduces to pure $SU(M)$ SYM + massless modes (“Goldstone” supermultiplet) and confines



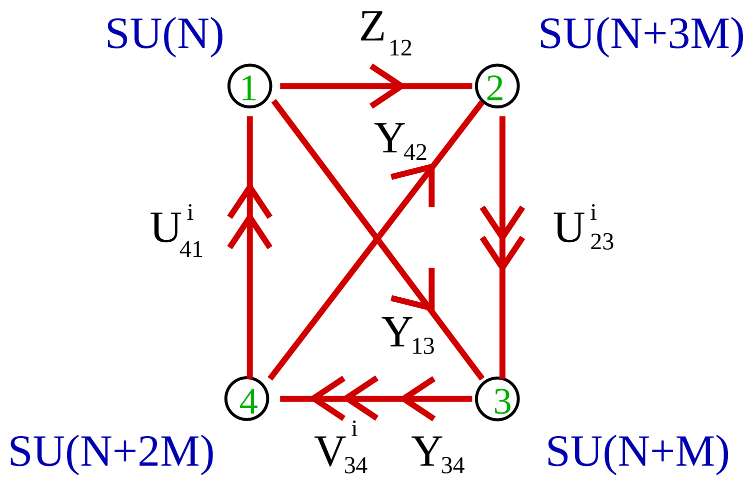
Note: couplings of Goldstone mode are suppressed by $1/\Lambda_{2M}$ and at weak coupling $g_s M \to 0$, where the scale are well separated, this amounts to complete decoupling from IR physics.





$\longrightarrow$ conifold fractional branes are deformation fractional branes: theory confines and admits M susy vacua.

Back to our case. Take $Y^{2,1}$ for simplicity. The (modified) quiver reads



At leading order, the deformed β-functions read (recall: $\beta_{1/g^2} \sim -\beta_g$)

$$b_2 = M(10 - \sqrt{13}) = -b_1 > 0 \ , \ b_4 = M(7\sqrt{13} - 22) = -b_3 > 0$$

Note: similarly to the conifold, one can show the flow is best described by a cascade of Seiberg dualities: $N \to N - M \to N - 2M \to \ldots$

Crucial is the self-similarity of superpotential upon duality transformations. The cascade continues until $N_f \leq N_c$ for some node: there the moduli space gets modified and the theory is not self-similar anymore.

What about dual sugra background? Expectations: metric modified, dilaton still constant, F_3 and H_3 turned on, running F_5-flux.

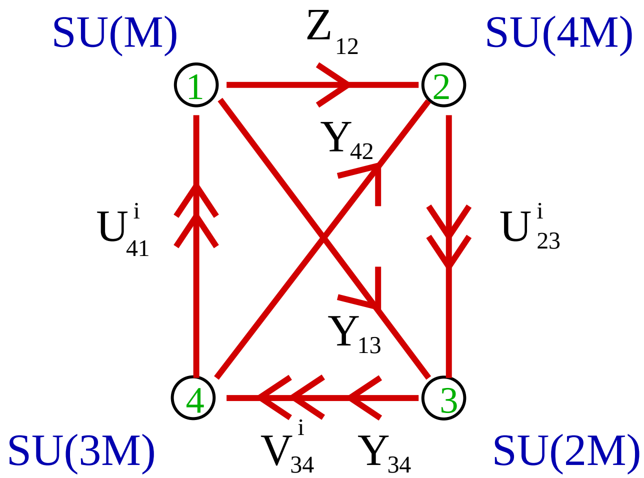
A solution having these properties exists [Herzog-Ejaz-Klebanov] but it is singular at short distances ($\sim$ IR of the gauge theory).

However, as the conifold KT solution, it encodes all UV physics: e.g. running of gauge couplings and decreasing of effective d.o.f. exactly match the duality cascade

$$\int_{S^2} B_2 \sim M \ \log(r/r_0) \ , \ \int_{Y^{p,q}} F_5 \sim f(r)$$

Is there any regular background exactly encoding the IR physics? Still not found. In fact, there are geometric arguments [Altmann] ruling out the existence of a (supersymmetric) deformed background! No confinement in here?

Choose $N = kM$. After $k-1$ cascade steps the effective number of regular branes reduces to M



Node 2 has $N_f = N_c$ and the quantum moduli space changes. The effective description of $SU(4M)$ low energy dynamics is given in terms of gauge invariant d.o.f., mesons and baryons, subject to the quantum constraint

$$\det \mathcal{M} - \mathcal{B}\tilde{\mathcal{B}} - \Lambda^{8M} = 0$$

The meson matrix is given by

$$\mathcal{M} = \left(\mathcal{M}^i_{13}, \mathcal{M}^i_{43}\right) \quad \text{with} \quad \mathcal{M}^i_{13} = Z_{12} U^i_{23} \;\; , \;\; \mathcal{M}^i_{43} = Y_{42} U^i_{23}$$

while baryons are schematically

$$\mathcal{B} \sim \left(U^i_{23}\right)^{4M} \qquad \tilde{\mathcal{B}} \sim \left(Z_{12}\right)^{2M} \left(Y_{42}\right)^{2M}$$

The superpotential W reads

$$\begin{aligned} W \quad &= \quad \mathrm{Tr}\left[\epsilon_{ij} V^i_{34} U^j_{41} Y_{13} + \epsilon_{ij} \mathcal{M}^i_{43} V^j_{34} + \epsilon_{ij} Y_{34} U^i_{41} \mathcal{M}^j_{13}\right] + \\ & \quad + \xi\left(\det \mathcal{M} - \mathcal{B}\tilde{\mathcal{B}} - \Lambda^{8M}\right) \end{aligned}$$

where ξ is a Lagrangian multiplier implementing the quantum constraint.

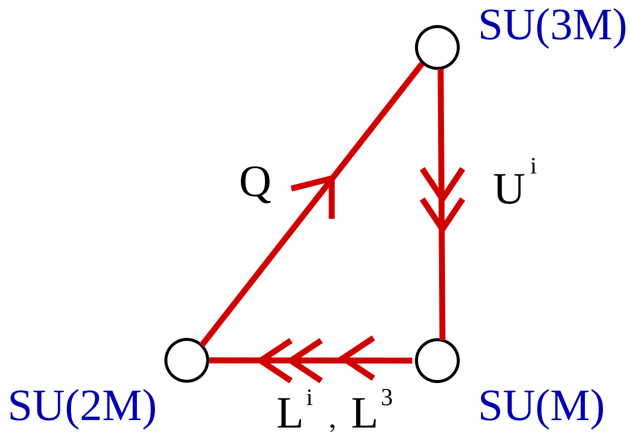
The F-term eqs. for ξ, $\mathcal{B}$ and $\tilde{\mathcal{B}}$ have two possible solutions:

- The mesonic branch: $\mathcal{B} = \tilde{\mathcal{B}} = 0$, $\det \mathcal{M} = \Lambda^{8M}$. The F-term eqs. for V_{34}^i are

$$\mathcal{M}_{43}^i = U_{41}^i Y_{13}$$

that, after fixing the D-term eqs., give $\mathcal{M}_{43}^1 = 0 \longrightarrow \det \mathcal{M} = 0$. Hence the mesonic branch is empty.

- The baryonic branch, $\xi = \det \mathcal{M} = 0$, $\mathcal{B}\tilde{\mathcal{B}} = -\Lambda^{8M}$. The mesons $\mathcal{M}_{43}^i$ are massive and can be integrated out. In the IR the theory reduces to



Note: this is the analogue of the single dot in the conifold theory!

The only gauge invariants are baryons constructed out of L_i and L_3, and mesons constructed out of Q and U^i

$$\mathcal{M} = (M^1, M^2) \text{ with } M^i = QU^i \ , \ \mathcal{B} \sim (L^\alpha)^{2M} \text{ with } \alpha = 1,2,3$$

The $SU(3M)$ factor has $N_f < N_c$ and the superpotential includes an ADS term contribution

$$W = \text{Tr}\left[M^1 L^2 - M^2 L^1\right] + M \left(\frac{\Lambda^{7M}}{\text{det}\,\mathcal{M}}\right)^{1/M}$$

... but this superpotential admits no susy vacua! In fact, the baryon singlet

$$\mathcal{B}_{|0,0>} \sim \epsilon_{a_1 \dots a_{2M}} \epsilon^{i_1 \dots i_M} \epsilon^{j_1 \dots j_M} (L^1)^{a_1}_{i_1} \dots (L^1)^{a_M}_{i_M} (L^2)^{a_{M+1}}_{j_1} \dots (L^2)^{a_{2M}}_{j_M}$$

is a runaway direction. [Berenstein et al., Franco et al., Bertolini-Bigazzi-Cotrone]

This can be generalized to the full $Y^{p,q}$ series: the baryon singlet is always runaway. [Intriligator-Seiberg, Brini-Forcella, Butti]

Conclusion: $Y^{p,q}$ fractional branes are SB branes!

Question: Can we find some deformation in order to avoid runaway behavior and get truly (meta)-stable non susy vacua?

- First possibility is suggested by DSB model building: adding tree level baryon coupling $\sim \lambda^{j,m} B_{|j,m>}$ may lift classical flat directions (cfr. N/N-1 model). However, a preliminary analysis suggests negative results for generic number of fractional branes: for *any* choice of the baryon coupling matrix, at least *one* classical direction is unlifted.

- Second possibility is to cook-up a modified quiver where DSB is put *by hand*, generalizing the 3-2 model. ABJ-anomaly vanishing conditions and *educated* dynamics seem incompatible.

- Leave the realm of toric varieties?

- ...

Work is in progress along these lines.

◇ **CONCLUSIONS**

- The $Y^{p,q}$ manifolds provide a new class of viable $\mathcal{N}=1$ dual pairs for which remarkable checks for AdS/CFT and a-maximization conjectures have been performed.

- Addition of fractional branes gives surprises: physics different from the conifold case, supersymmetry is broken! Regular solution still unknown but stable (or meta-stable) vacuum seems to be excluded.

- What the structure of a sugra solution dual to runaway behavior? Can we deform the gauge theory so to lift classical flat directions? Note that a drastic (UV!) deformation of the geometry seems to be needed.