ASIMMETRIA LEPTONICA INDOTTA DALL'ACCOPPIAMENTO SPIN-GRAVITY

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- An open issue of modern Cosmology and particle physics is the origin of the *baryon number asymmetry*, i.e. as far into the Universe as we can see, there is an excess of matter over anti-matter.
- The parameter characterizing such a asymmetry is $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$
- The observation of acoustic peaks in the CMB combined with measurements of large scale structures has recently led to $\eta^{CMB} \sim (6.3 \pm 0.3) \times 10^{-10}$, at $T_{CMB} \sim 1eV$, $_{CMB} \sim 10^{13} sec$ for a FRW Universe.
- An independent determination of η is obtained from BBN, $\eta^{BBN} \sim (3.2 - 6.9) \times 10^{-10}$, at $T_{BBN} \sim 1MeV$, $t_{BBN} \sim 10 sec$.
- The consistence of η^{CMB} and η^{BBN} is an impressive test of the cosmological standard model. Nevertheless, the standard cosmological model cannot explain the observed value of η.

Conventionally, it is argued that the baryon asymmetry is (dynamically) generated from an initial baryon symmetric phase as long as the following conditions are satisfied (Sakharov's conditions)

- Interactions that do not conserve baryon number;
- C and CP violation;
- The existence of non-equilibrium processes (out of thermal equilibrium).

Nevertheless, if CPT is violated the baryon number asymmetry can be generated in thermal equilibrium (Cohen, Kaplan, 1987).

We shall work in the context of Leptogenesis scenario. Lepton asymmetry can be converted into baryon asymmetry via electroweak effects madiated by sphalerons. Thus even if the baryon number is conserved at high energy scales, it is possible to generate the baryon number asymmetry in the present Universe if lepton asymmetry is generated at either GUT or intermediate scales (Fukugita, Yanagida, 1986).

Idea of the paper: CPT is violated spontaneously due to the spin-connection couplings of fermions with cosmological gravitational waves. It is well known that inflation generates a nearly scale invariant spectrum of gravitational waves. The spin connection couplings split the energy levels of neutrinos compared to anti-neutrinos and in presence of lepton number violating interaction there is a net asymmetry generated between neutrinos and anti-neutrinos at *thermodynamic equilibrium*. Lepton number violation is generated by the dimension five operator introduced by Weinberg which also generates the neutrino masses after electro-weak symmetry breaking. The lepton-asymmetry gets frozen-in when the lepton-number violating processes decouple. Baryon asymmetry can then be generated from this lepton-asymmetry by the electroweak sphaleron processes. Sphaleron processes conserve $(B-L)$ so a lepton asymmetry generated in the GUT era can be converted to baryon asymmetry of the same magnitude (G. Lambiase, S. Mohanty, A.R. Prasanna, 2006).

The main topics for carried out calculations are:

- The general covariant coupling of spin 1/2 particles to gravity is given by the Lagrangian

$$\mathcal{L} = \sqrt{-g}(\bar{\psi}\gamma^a D_a \psi - m\bar{\psi}\psi)\,. \qquad (1)$$

  $D_a = \partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}$ is the covariant derivative, and ω_{bca} are the spin-connections $\omega_{bca} = e_{b\lambda}\left(\partial_a e^{\lambda}{}_c + \Gamma^{\lambda}_{\gamma\mu} e^{\gamma}_c e^{\mu}_a\right)$, $\sigma^{bc} = \frac{i}{2}[\gamma^b,\ \gamma^c]$ are generators of tangent space Lorentz transformation.

- the identity $\gamma^a\gamma^b\gamma^c = \eta^{ab}\gamma^c + \eta^{bc}\gamma^a - \eta^{ac}\gamma^b - i\epsilon^{abcd}\gamma_5\gamma_d$

- the spin connection term can be reduced to a vector $A^a\gamma_a$ and an axial vector $iB^d\gamma_5\gamma_d$. The vector term turns out to be anti-hermitian and disappears when the hermitian conjugate part is added to the lagrangian. The surviving interaction term which describes the spin-connection coupling of fermions to gravity can be written as a axial-vector $\mathcal{L} = \det(e)\ \bar{\psi}\left(\ i\gamma^a\partial_a\ -\ m\ -\ \gamma_5\gamma_d B^d\ \right)\psi$, $B^d = \epsilon^{abcd} e_{b\lambda}(\partial_a e^{\lambda}_c + \Gamma^{\lambda}_{\alpha\mu} e^{\alpha}_c e^{\mu}_a)$.

In a local inertial frame of the fermion, the effect of a gravitational field appears as a axial-vector interaction term shown in L. We now calculate the four vector B^d for a perturbed Robertson-Walker universe.

The general form of perturbations on a flat Robertson-Walker expanding universe can be written as

$$ds^2 = a(\tau)^2[(1+2\phi)d\tau^2 - \omega_i dx^i d\tau - ((1+2\psi)\delta_{ij} + h_{ij})dx^i dx^j]\,, \tag{2}$$

where ϕ and ψ are scalar, ω_i are vector and h_{ij} are the tensor fluctuations of the metric. For our application we need only the tensor perturbations and we choose the transverse-traceless (TT) gauge $h^i_i = 0, \partial^i h_{ij} = 0$ for the tensor perturbations. In the TT gauge the perturbed Robertson-Walker can be expressed as

$$g_{\mu\nu} = a(\tau)^2 \begin{pmatrix} 1+2\phi & -\omega_1 & -\omega_2 & -\omega_3 \\ -\omega_1 & -(1+2\psi)+h_+ & h_\times & 0 \\ -\omega_2 & h_\times & -(1+2\psi)-h_+ & 0 \\ -\omega_3 & 0 & 0 & -(1+2\psi) \end{pmatrix} \tag{3}$$

An orthogonal set of vierbiens e^a_μ for this metric is given by

$$e^a_\mu = a(\tau) \begin{pmatrix} 1+\phi & -\omega_1 & -\omega_2 & -\omega_3 \\ 0 & -(1+\psi)+h_+/2 & h_\times & 0 \\ 0 & 0 & -(1+\psi)-h_+/2 & 0 \\ 0 & 0 & 0 & -(1+\psi) \end{pmatrix} \tag{4}$$

The expression for the components of the four vector field B^d is given by

$$B^0 = \partial_3 h_\times\,, \quad B^1 = (\bigtriangledown \times \vec{\omega})^1\,, \quad B^2 = (\bigtriangledown \times \vec{\omega})^2\,, \quad B^3 = (\bigtriangledown \times \vec{\omega})^3 + \partial_\tau h_\times\,. \tag{5}$$

The fermion bilinear term $\bar{\psi}\gamma_5\gamma_a\psi$ is odd under CPT transformation. When one treats B^a as a background field then the interaction term in L explicitly violates CPT. When the primordial metric fluctuations become classical, i.e there is no back-reaction of the micro-physics involving the fermions on the metric and B^a is considered as a fixed external field, then CPT is violated spontaneously.

The gravitational spin connection coupling for the neutrinos at high energy is given by

$$\begin{aligned}\mathcal{L} &= \det(e)[(i\bar{\nu}_L\gamma^a\partial_a\nu_L + i\bar{\nu}_R\gamma_a\partial_a\nu_R) \\ &+ m\bar{\nu}_L\nu_R + m^\dagger\bar{\nu}_R\nu_L + B^a(\bar{\nu}_R\gamma_a\nu_R - \bar{\nu}_L\gamma_a\nu_L)]\,,\end{aligned} \tag{6}$$

If we consider only the Standard Model fermions then the right handed neutrinos carry the opposite Lepton number compared to the left handed neutrinos, $\nu_R = (\nu_L)^c$ and the mass term in (6) is of the Majorana type (we have suppressed the generation index).

The dispersion relation of left and right helicity neutrinos fields are given by

$$\eta^{ab}(p_a + \xi B_a)(p_b + \xi B_b) = m^2\,, \tag{7}$$

where $\xi = -1$ for ν_L and $\xi = 1$ for ν_R. Keeping terms linear in the perturbations B^a, the free particle energy of the left and right helicity states is

$$E_{L,R}(p) = p + \frac{m^2}{2p} \mp \left(B_0 - \frac{\mathbf{p}\cdot\mathbf{B}}{p}\right)\,, \tag{8}$$

with $p = |\mathbf{p}|$. In the Standard Model ν_L carry lepton number $+1$ and ν_R are assigned lepton number (-1). In the presence of non-zero metric fluctuations, there is a split in energy levels of $\nu_{L,R}$. If there are GUT processes that violate lepton number freely above some decoupling temperature T_d, then the equilibrium value of lepton asymmetry generated for all $T > T_d$ will be

$$n(\nu_L) - n(\nu_R) = \frac{g}{2\pi^2}\int d^3p\left[\frac{1}{1+e^{\frac{E_L}{T}}} - \frac{1}{1+e^{\frac{E_R}{T}}}\right] \tag{9}$$

In the ultra-relativistic regime $p \gg m_\nu$ and assuming that $B_0 \ll T$, the expression (9) for lepton asymmetry reduces to

$$\Delta n_L = \frac{gT^3}{6}\left(\frac{B_0}{T}\right)\,. \tag{10}$$

The dependence on $\mathbf{B}$ drops out after angular integration in (9) and the lepton asymmetry depends on the tensor perturbations only through B^0.

Spectrum of the GWs

To compute the spectrum of gravitational waves $h(\mathbf{x},\tau)$ during inflation, we express express $h_\times$ in terms of the creation- annihilation operator

$$h(\mathbf{x},\tau)=\frac{\sqrt{16\pi}}{aM_p}\int\frac{d^3k}{(2\pi)^{3/2}}\left(a_{\mathbf{k}}\ f_k(\tau)+a^\dagger_{-\mathbf{k}}\ f^*_k(\tau)\right)\ e^{i\mathbf{k}\cdot\mathbf{x}} \tag{11}$$

where $a(\tau)$ is the scale factor, $\mathbf{k}$ is the comoving wavenumber, $k=|\mathbf{k}|$, and $M_p=1.22\ 10^{19}GeV$ is the Planck mass. The mode functions $f_k(\tau)$ obey the minimally coupled Klein-Gordon equation

$$f''_k+\left(k^2-\frac{a''}{a}\right)f_k=0\,. \tag{12}$$

During de Sitter era, the scale factor $a(\tau)=-1/(H_I\tau)$ where H_I is the Hubble parameter, and Eq. (12) has the solution

$$f_k(\tau)=\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1-\frac{i}{k\tau}\right)\,, \tag{13}$$

which matches the positive frequency "flat space" solutions $e^{-ik\tau}/\sqrt{2k}$ in the limit of $k\tau\gg1$. The two point correlation of gravitational waves generated by inflation is

$$\langle h(\mathbf{x},\tau)h(\mathbf{x},\tau)\rangle^{inf}\equiv\int\frac{dk}{k}\,(|h_k|^2)^{inf}\,, \tag{14}$$

with the spectrum of gravitational waves given by the scale invariant form

$$(|h_k|^2)^{inf}=\frac{4}{\pi}\frac{H_I^2}{M_p^2}\,. \tag{15}$$

In the radiation era, $a(\tau)\sim\tau$ and the equation for f_k gives plane wave solutions $f_k=(1/\sqrt{2k})exp(-k\tau)$. Therefore in the radiation era the amplitudes of h redshifts as a^{-1}. The gravitational waves inside the horizon in the radiation era will be

$$h_k^{rad}=h_k^{inf}\,\frac{a_k}{a(\tau)}=h_k^{inf}\,\frac{T}{T_k}\,, \tag{16}$$

where h_k^{inf} are the gravitational waves generated by inflation, a_k and T_k are the scale factor and the temperature when the modes of wavenumber k

entered the horizon in the radiation era. The horizon entry of mode k occurs when

$$\frac{a_k H_k}{k} = \frac{a(T)\, T\, H_k}{T_k\, k} = 1\,, \tag{17}$$

where $H_k = 1.67\sqrt{g_*}T_k^2/M_p$ is the Hubble parameter at the time of horizon crossing of the k the mode (g_* is the number of relativistic degrees of freedom which for the Standard Model is $g_* = 106.7$). Solving equation (17) for T_k we get

$$T_k = \frac{1}{1.67\sqrt{g_*}} \frac{k\, M_p}{a(\tau)\, T}\,. \tag{18}$$

The amplitude of the gravitational waves of mode k inside the radiation horizon is, using (18) and (16), given by

$$h_k^{rad} = h_k^{inf}\, \frac{a(T)}{k}\, \frac{T^2\, 1.67\, \sqrt{g_*}}{M_p}\,. \tag{19}$$

Note that the gravitational wave spectrum inside the radiation era horizon is no longer scale invariant. The gravitational waves in position space have the correlation function

$$\langle h(\mathbf{x},\tau) h(\mathbf{x},\tau)\rangle^{rad} = \int \frac{dk}{k}\, (h_k^{rad})^2\,, \tag{20}$$

hence for the spin connection B^0 generated by the inflationary gravitational waves in the radiation era, we get the r.m.s value of spin connection that determines the lepton asymmetry is $(B_0)_{rms} = \sqrt{\langle B_0^2 \rangle}$,

$$(B_0)_{rms} = \frac{2}{\sqrt{\pi}} \left(\frac{H_I}{M_p^2}\, T^2\, 1.67\, \sqrt{g_*} \right) \sqrt{N}\,. \tag{21}$$

where is the total e-folding of the scale factor during the Inflation ($N \sim 50 - 70$).

Lepton Asymmetry

The lepton asymmetry as a function of temperature can therefore be expressed as (taking $g = 3$ for the three neutrino flavors)

$$\Delta n_L(T) = \frac{1}{\sqrt{\pi}}\,(1.67\,\sqrt{g_*})\,\sqrt{N}\left(\frac{T^4\,H_I}{M_p^2}\right)\,. \tag{22}$$

The lepton number to entropy density ($s = 0.44\ g_*\ T^3$) turns out to be independent of temperature and is given by

$$\Delta L \equiv \frac{\Delta n_L(T)}{s(T)} \simeq 2.14\,\frac{T\,H_I\,\sqrt{N}}{M_p^2\,\sqrt{g_*}}\,. \tag{23}$$

Lepton number asymmetry will be generated as long as the lepton number violating interactions are in thermal equilibrium. Once these reactions decouple at some decoupling temperature T_d, which we shall determine, the $\Delta n_L(T)/s(T)$ ratio remains fixed for all $T < T_d$.

To calculate the decoupling temperature of the lepton number violating processes we turn to a specific effective dimension five operator which gives rise to Majorana masses for the neutrinos introduced by Weinberg (Weinberg, 1979)

$$\mathcal{L}_W = \frac{C_{\alpha\beta}}{2M}(\overline{l_{L\alpha}{}^c}\ \tilde{\phi}^*)(\tilde{\phi}^\dagger\ l_{L\beta}) + h.c. \tag{24}$$

where $l_{L\alpha} = (\nu_\alpha, e^-_\alpha)^T_L$ is the left-handed lepton doublet (α denotes the generation), $\phi = (\phi^+, \phi^0)^T$ is the Higgs doublet and $\tilde{\phi} \equiv i\sigma_2\phi^* = (-\phi^{0*}, \phi^-)^T$. M is some large mass scale and $C_{\alpha\beta}$ are of order unity.

The $\Delta L = 2$ interactions that result from the operator (24) are

$$\begin{aligned} \nu_L + \phi^0 &\longleftrightarrow \nu_R + \phi^0\,, \\ \nu_R + \phi^{0*} &\longleftrightarrow \nu_L + \phi^{0*}\,. \end{aligned} \tag{25}$$

In the absence of the gravitational waves the forward reactions would equal the backward reactions and no net lepton number would be generated. In the presence of a background gravitational waves the energy levels of the left and right helicity neutrinos are no longer degenerate and this leads to a difference in the number density of left and right handed neutrinos of the magnitude given by equation (10) at thermal equilibrium. This process continues till the interactions (25) decouple. The decoupling temperature is estimated as follows. The cross section for the interaction $\nu_{L\alpha} + \phi^0 \longleftrightarrow \nu_{R\beta} + \phi^0$ is

$$\sigma = \frac{|C_{\alpha\beta}|^2}{M^2}\frac{1}{\pi}\,, \tag{26}$$

and interaction rate $\Gamma = \langle n_\phi\ \sigma\rangle$ of the $\Delta L = 2$ interactions is

$$\Gamma = \frac{0.122}{\pi}\frac{|C_{\alpha\beta}|^2 T^3}{M^2}\,. \tag{27}$$

In the electroweak era, when the Higgs field in (24) acquires a *vev*, $\langle\phi\rangle = (0, v)^T$ (where $v = 174\ GeV$), this operator gives rise to a neutrino mass matrix $m_{\alpha\beta} = \frac{v^2 C_{\alpha\beta}}{M}$. We can therefore substitute the couplings $\frac{C_{\alpha\beta}}{M}$ in terms of the light left handed Majorana neutrino mass. At the decoupling temperature the interaction rate $\Gamma(T)$ falls below the expansion rate $H(T) = 1.7\sqrt{g_*}\ T^2/M_p$. The decoupling temperature is obtained from equation $\Gamma(T_d) = H(T_d)$ and turns out to be

$$T_d = 13.68\pi\ \sqrt{g_*}\ \frac{v^4}{m_\nu^2 M_p}\,, \tag{28}$$

where m_ν is the mass of the heaviest neutrino. A lower bound on the mass of the heaviest neutrino is given by atmospheric neutrino experiments $m_\nu^2 > \Delta_{atm} = 2.5\ 10^{-3} eV^2$ (SuperKAmiokande, 2005) which means that the decoupling temperature has an upper bound given by $T_d = 1.3\ 10^{13}(\Delta_{atm}/m_\nu^2)\, GeV$. Substituting the expression (28) for T in (23), we finally obtain the formula for lepton number

$$\begin{aligned} L &= 92.0 \left(\frac{v^4\, H_I}{m_\nu^2\, M_p^3}\right)\sqrt{N} \\ &= 7.4\, 10^{-11} \frac{H_I}{4\, 10^{14} GeV}\, \frac{2.5\, 10^{-3} eV^2}{m_\nu^2}\, \frac{\sqrt{N}}{10}\,. \end{aligned}$$

So we see if inflation takes place at the GUT scale ($4\, 10^{16} GeV$), where $H_I \sim 4\, 10^{14} GeV$ which is allowed by CMB (Pilo, Riotto, Zaffaroni, 2004; Knox, Song, 2002), if there are about $N \sim 100$ e-foldings of the scale factor, and if the neutrinos have hierarchical masses with the heaviest neutrino given by $\sqrt{\Delta_{atm}}$, we generate the required lepton asymmetry.