Avoiding the Final Singularity

Holographic and Semiclassical Models of Gravitational Collapse

Roberto Casadio
University of Bologna

Vietri sul Mare
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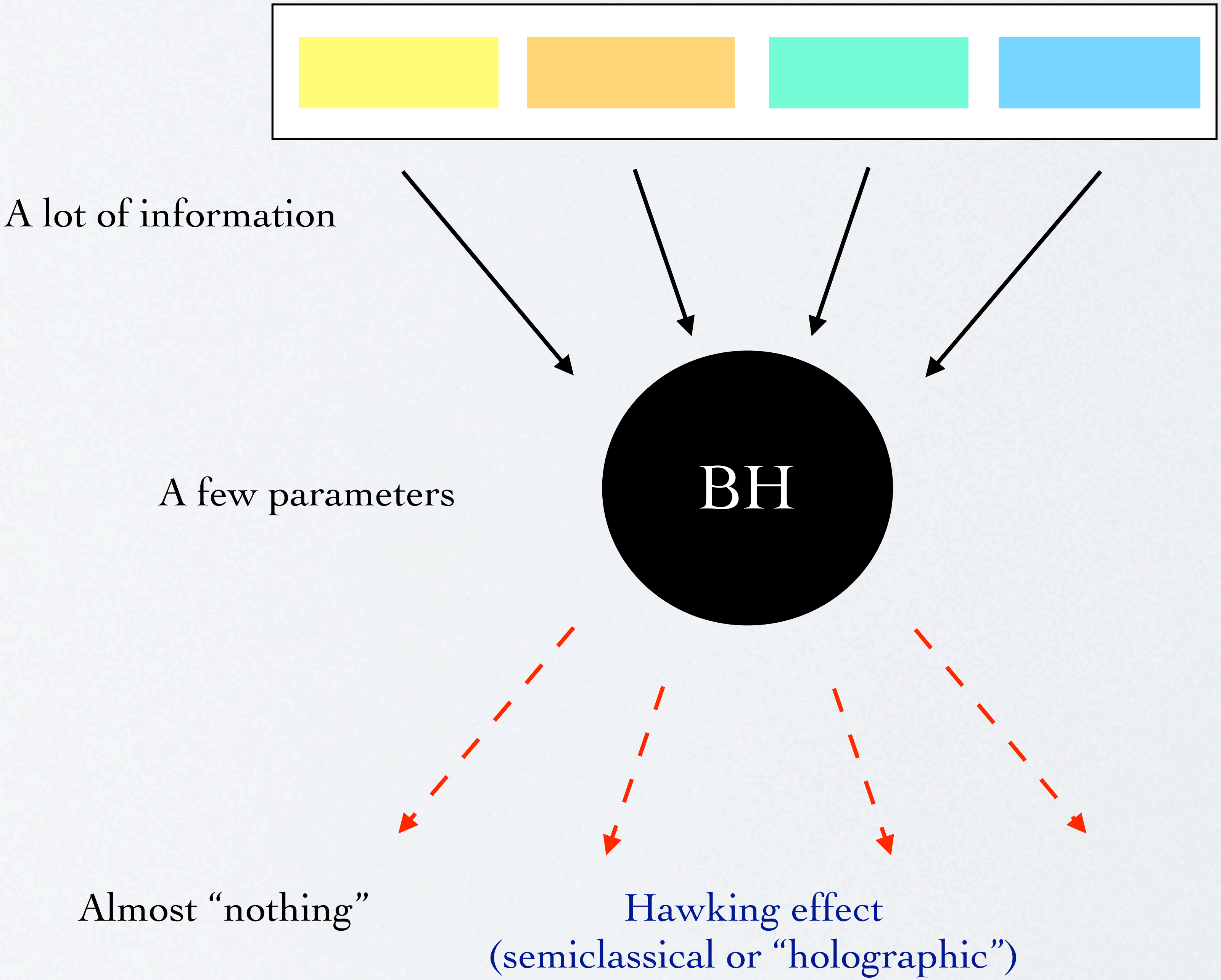
The gravitational collapse of a spherically symmetric (homogeneous) dust star is analyzed in a brane-world model [1] and in the semiclassical limit of the Wheeler-deWitt theory [2]. These two (unrelated?) approaches suggest that the central singularity does not form, in contrast with what is predicted by (classical) General Relativity.

[1] R. C. and C. Germani, Prog. Theor. Phys. 114 (2005) 23 [arXiv:hep-th/0407191]
[2] R. C., Int. J. Mod. Phys. D 9 (2000) 511 [arXiv:gr-qc/9810073].

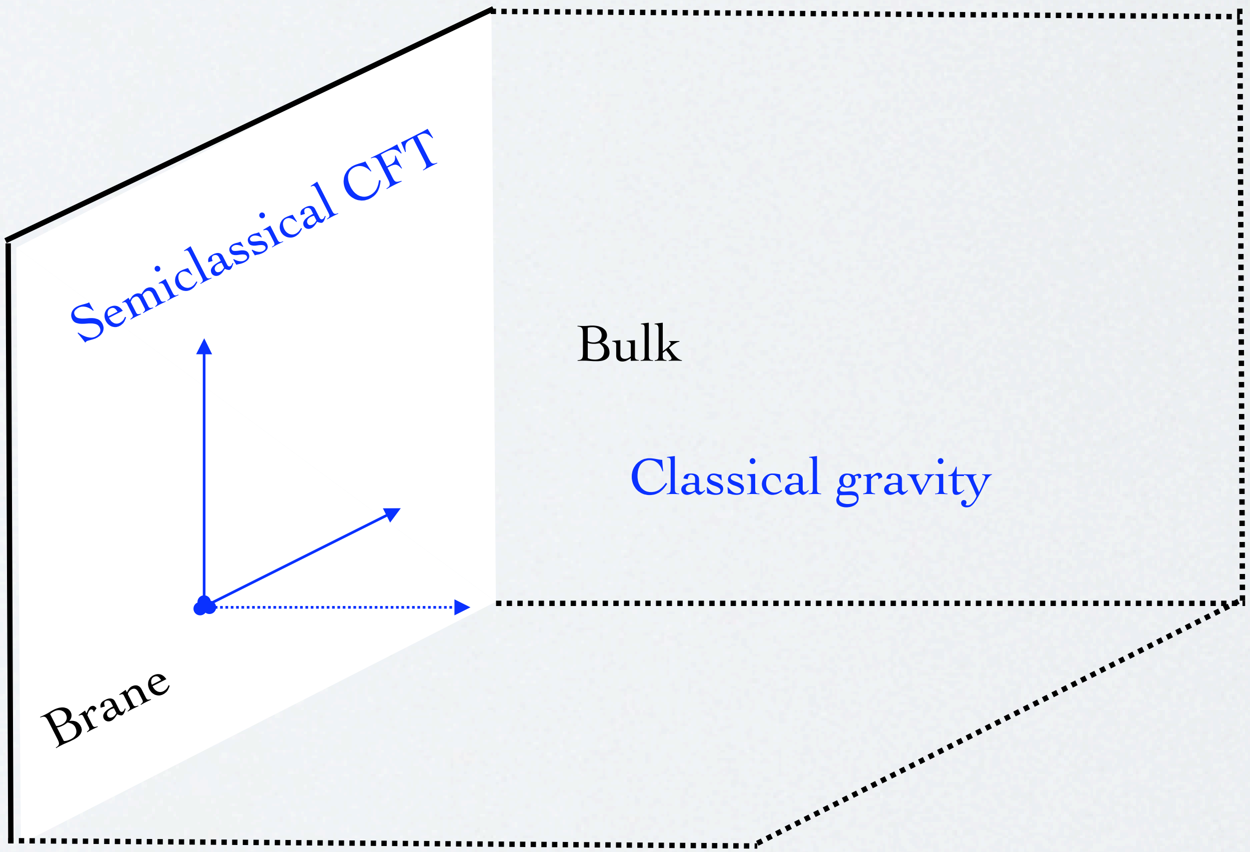
Plan of the talk

1. The problem: Hawking radiation and "unitarity"
2. BW gravitational collapse:
   - 2.i General (hydrodynamical) equations
   - 2.ii Spherically symmetric dust
   - 2.iii BW-corrected OS model:
     - Black hole formation and evaporation
     - Horizon and luminosity
     - Trace anomaly
3. Semiclassical gravitational collapse:
   - 3.i BO WDW equation
   - 3.ii WKB-corrected OS model
     - Core bounce
4. Conclusions

Hawking radiation and “unitarity”



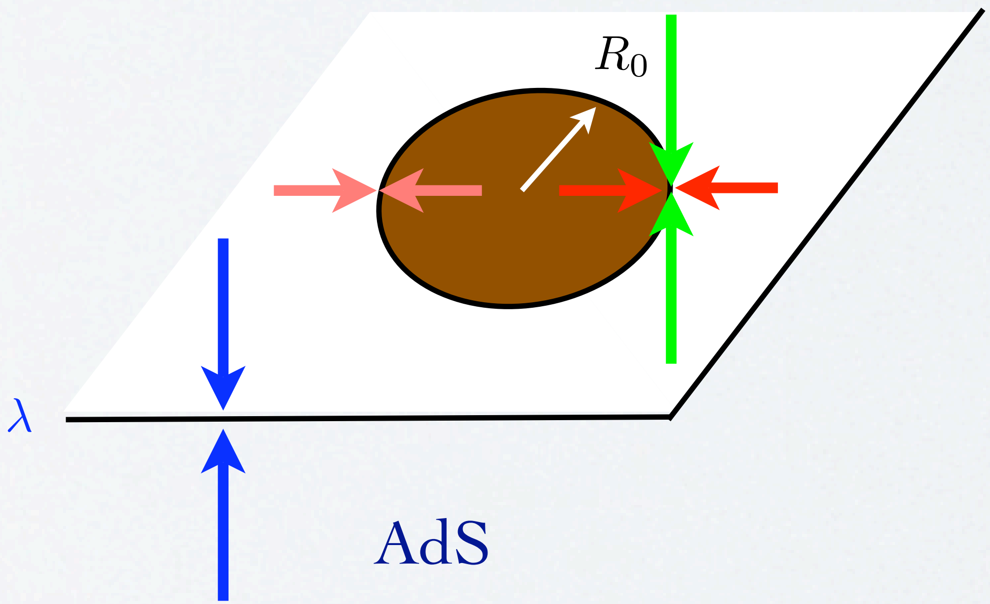
Simple BW picture

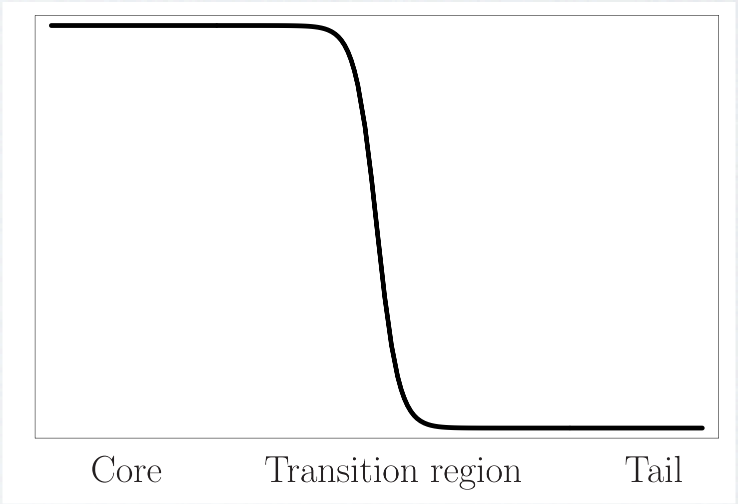


Semiclassical Hawking effect ⟷ Classical bulk dynamics

Simple BW picture

Double junction conditions → "atmosphere"





Modified brane equations → "Hawking radiation"

$$\dot{R}_0^2 = \frac{2\,M_{\rm S}}{R_0} + \frac{3}{4\,\pi\,\lambda\,R_0^2}\left(\frac{M_{\rm S}^2}{R_0^2} - \frac{3}{4}\,\mu\right)$$

$$\dot{M}_{\rm H} \simeq -\frac{9}{128\,\pi\,\lambda}\,\frac{\mu - 1}{M_{\rm H}^2}$$

$$M = M_{\rm S} + \frac{1}{\lambda}\,m(R)$$

$$R_0(\tau_{\rm H}) = 2\,M(\tau_{\rm H})$$

General (hydrodynamical) equations: $G_{\mu\nu} = 8\,\pi\,T^{\rm eff}_{\mu\nu}$

$$T^{\rm eff}_{\mu\nu} = \rho^{\rm eff}\,u_\mu\,u_\nu + p^{\rm eff}\,h_{\mu\nu} + q^{\rm eff}_{(\mu}\,u_{\nu)} + \pi^{\rm eff}_{\mu\nu}$$

$$u^\mu u_\mu = -1$$

$$h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

$$-\frac{1}{8\pi}\,\mathcal{E}_{\mu\nu} = \mathcal{U}\left(u_\mu\,u_\nu + \frac{1}{3}\,h_{\mu\nu}\right) + Q_\mu\,u_\nu + Q_\nu\,u_\mu + \Pi_{\mu\nu}$$

$$\rho^{\rm eff} = \rho\left(1 + \tfrac{\rho}{2\,\lambda} + \tfrac{\mathcal{U}}{\rho}\right)$$

$$p^{\rm eff} = p + \tfrac{\rho}{2\,\lambda}\,(2\,p + \rho) + \tfrac{\mathcal{U}}{3}$$

$$q^{\rm eff}_\mu = Q_\mu$$

$$\pi^{\rm eff}_{\mu\nu} = \Pi_{\mu\nu}$$

LCE
$$\begin{cases} \dot\rho + \Theta\,(\rho + p) = 0 \\ D_a p + (\rho + p)\,A_a = 0 \end{cases}$$

NLCE
$$\begin{cases} \dot{\mathcal{U}} + \tfrac{4}{3}\,\Theta\,\mathcal{U} + D^a\,Q_a + 2\,A^a\,Q_a + \sigma^{ab}\,\Pi_{ab} = 0 \\ \dot{Q}_a + \tfrac{4}{3}\,\Theta\,Q_a + \tfrac{1}{3}\,D_a\mathcal{U} + \tfrac{4}{3}\,\mathcal{U}\,A_a + D^b\Pi_{ab} \\ \quad + A^b\,\Pi_{ab} + \sigma_a{}^b\,Q_b - \omega_a{}^b\,Q_b = -\tfrac{\rho+p}{\lambda}\,D_a\rho \end{cases}$$

Spherically symmetric dust $(p = 0 \quad \wedge \quad \rho \neq 0 \quad \Longrightarrow \quad u^\alpha = (-1, 0, 0, 0))$

$$\rho^{\rm eff} = \rho\,\left(1 + \tfrac{\rho}{2\,\lambda}\right) + \mathcal{U}$$

$$p^{\rm eff} = \frac{\rho^2}{2\,\lambda} + \frac{\mathcal{U}}{3}$$

$$\pi^{\rm eff}_{\mu\nu} = \Pi_{\mu\nu}$$

→

Tolman geometry

$$ds^2 = -d\tau^2 + \left[R'(\tau,r)\right]^2\,dr^2 + R^2(\tau,r)\,d\Omega^2$$

with

$$\Pi^a{}_b = {\rm diag}\left(\frac{2}{3}\,\Pi, -\frac{1}{3}\,\Pi, -\frac{1}{3}\,\Pi\right)$$

↓

$$\partial_\tau \rho + \frac{\partial_\tau \partial_r\left(R^3\right)}{\partial_r\left(R^3\right)}\,\rho = 0$$

LCE →

$$\partial_\tau m_\rho = 0$$

"Bare" mass function:

$$m_\rho(r) \equiv \frac{4\,\pi}{3}\int_0^r \rho(\tau,x)\,\partial_x\left[R^3(\tau,x)\right]dx$$

↔

$$\rho(\tau,r) = \frac{m'_\rho}{4\,\pi\,R^2\,R'}$$

"Effective" mass function:

$$M(\tau, r) = \frac{4\pi}{3} \int_0^r \rho^{\mathrm{eff}}(\tau, x)\, \partial_x \left[R^3(\tau, x)\right] dx$$

$$G^{\tau}{}_{\tau} = -\frac{(\dot{R}^2\, R)'}{R^2\, R'} = -8\,\pi\,\rho^{\mathrm{eff}} \quad \longleftrightarrow \quad \dot{R}^2(\tau, r) = \frac{2\,M(\tau, r)}{R(\tau, r)}$$

$$\dot{M}(\tau, r) = \frac{4\pi}{3} \int_0^r \partial_\tau \left[\left(\frac{\rho^2}{2\,\lambda} + \mathcal{U}\right) \partial_x \left(R^3\right)\right] dx \neq 0$$

NLCE

$$\begin{cases} \dot{\mathcal{U}} + \frac{4}{3}\left[\frac{\dot{R}}{R}\left(2\mathcal{U} - \frac{\Pi}{2}\right) + \frac{\dot{R}'}{R'}\left(\mathcal{U} + \frac{\Pi}{2}\right)\right] = 0 \\ \frac{1}{3}\mathcal{U}' + \frac{2}{3}\left[\Pi' + 3\,\frac{R'}{R}\,\Pi\right] = -\frac{\rho}{\lambda}\,\rho' \end{cases}$$

→

In asymptotically flat BW:

$$\dot{\rho}' \neq 0 \quad \Longrightarrow \quad \Pi \neq 0$$

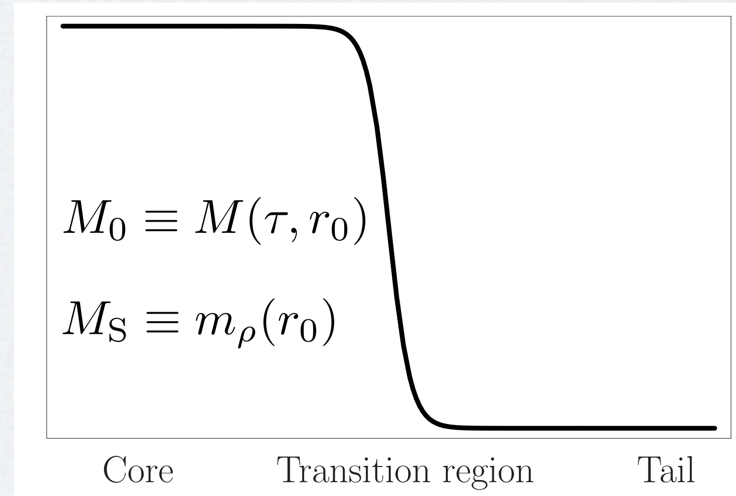
Corrected OS

1) The “core” $(\rho' \simeq 0 \implies \rho(\tau, r) = \rho_0(\tau))$

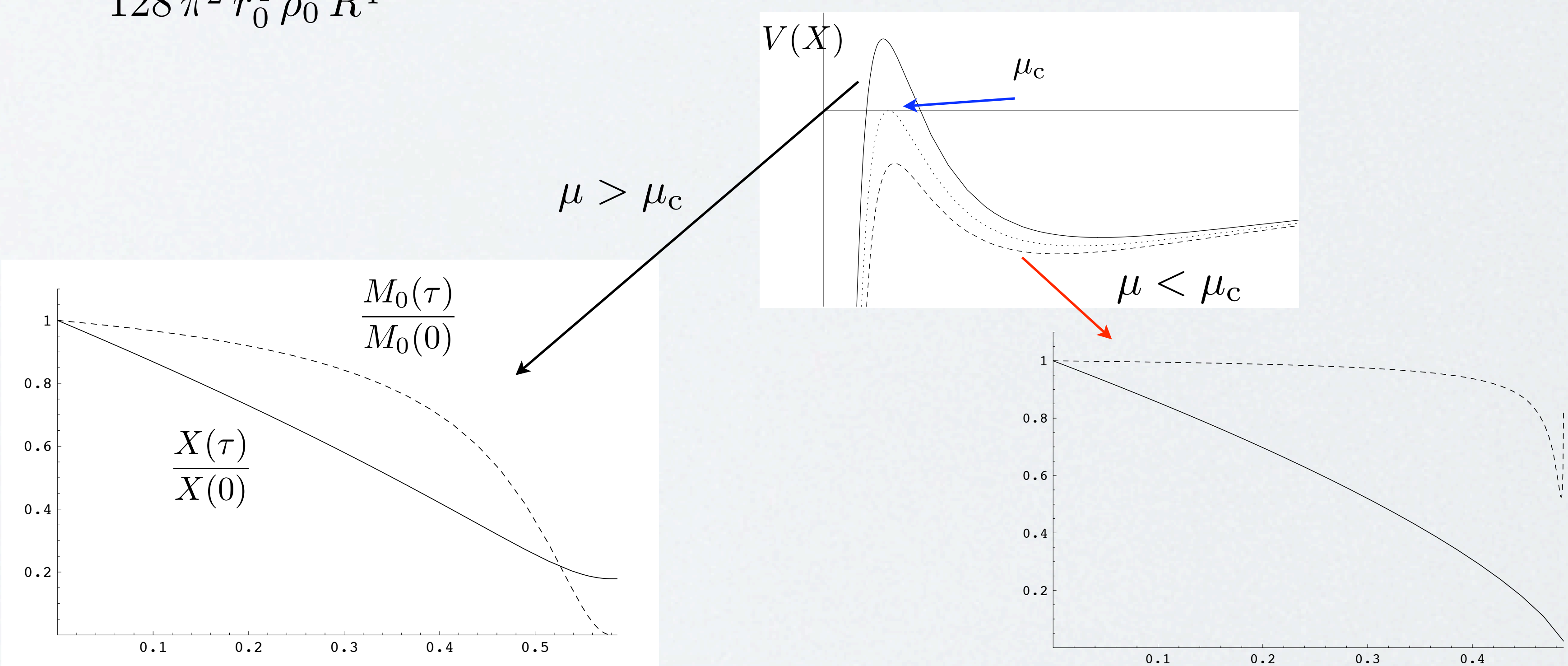
Perturbative parameter: $\epsilon \equiv \dfrac{\rho_0(0)}{\lambda}$

$$R(\tau, r) = \left(\frac{9}{2} M_{\rm S}\right)^{1/3} \frac{r}{r_0} X(\tau)$$

$$\mathcal{U} = -\frac{27\, \mu\, r^4\, \epsilon}{128\, \pi^2\, r_0^4\, \rho_0\, R^4}$$

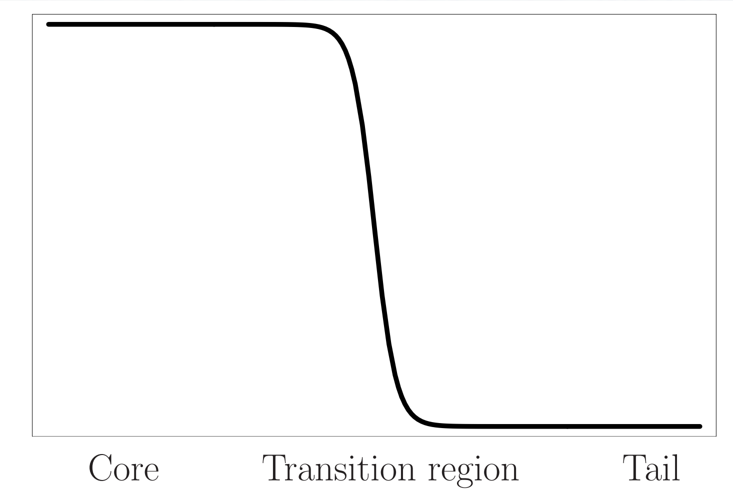


$0 \longleftrightarrow r_0$



Corrected OS

2) The "transition region" $(\rho' \neq 0 \implies \Pi \neq 0)$



$r_0 \longleftrightarrow r_s$

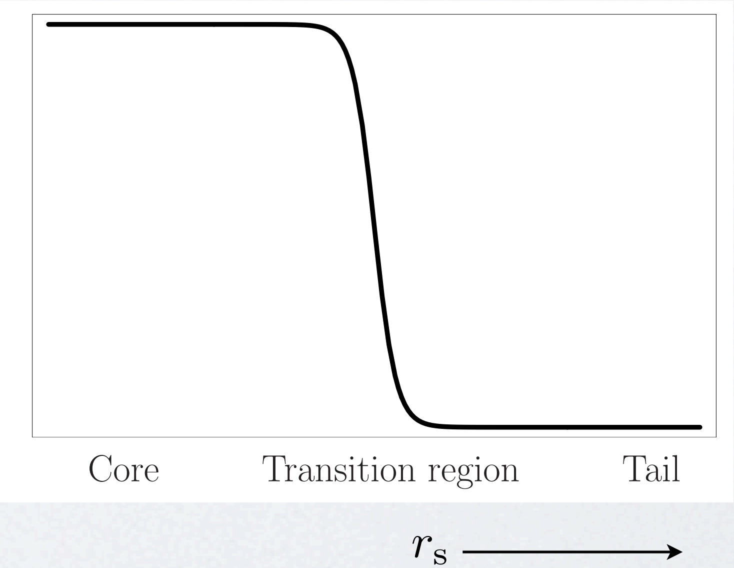
$$r_{\rm s} - r_0 = O(\epsilon) = m_\rho(r, r_0)$$

$$m_{\mathcal{U}}(\tau, r; r_0) = \frac{4\,\pi}{3} \int_{r_0}^{r} \mathcal{U}\,(R^3)'\,dx = O\left(\epsilon^2\right)$$

$$M(\tau, r) \simeq M_0(\tau) + m_\rho(r; r_0) + m_{\mathcal{U}}(\tau, r; r_0) \simeq M_0(\tau) + m_\rho(r; r_0)$$

$$\dot{M}(\tau, r) \simeq \dot{M}_0(\tau)$$

3) The "tail" ($\rho = O(\epsilon)$)



(a) $\lim_{r\to\infty} R(\tau, r) = \infty$ (infinite comoving frame)

(b) $\lim_{r\to\infty} M(\tau, r) < \infty \,, \quad \forall\, \tau > 0$ (asymptotic flatness)

$$\lim_{r\to\infty} \dot{M}(\tau, r) = 0 \,, \quad \forall\, \tau > 0$$

(total energy is conserved)

BH evolution

Apparent horizon: $\dot{R}(\tau, r_{\rm H}(\tau)) = -1$

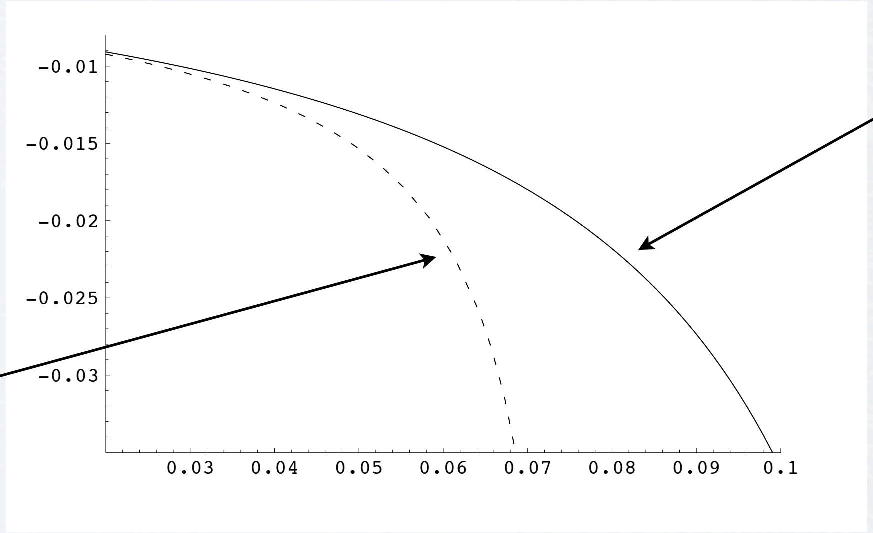
1) in the "core": it forms at $r = r_0$ at the time $\tau = \tau_{\rm H}^{\rm OS}$.

$$\frac{dM_{\rm H}}{d\tau} = \dot{M}_{\rm H} \equiv \dot{M}_0(\tau_{\rm H}^{\rm OS}) \simeq -\frac{9\,(\mu-1)\,\epsilon}{128\,\pi\,\rho_0\,M_{\rm H}^2}$$

2) in the "transition region":

$$\frac{dM_{\rm H}}{d\tau} \simeq \dot{M}_{\rm H}(\tau, r_{\rm H}(\tau)) \simeq \dot{M}_0(\tau) \simeq \dot{M}_0(\tau_{\rm H}^{\rm OS})$$

3) in the "tail":



Consistency bound:

$$\lambda > 10^{30}\ \text{GeV}$$

$$10^{-32}\ \text{mm} \ll \sqrt{\frac{3}{4\pi\,G\,\lambda}} \ll 10^{-9}\ \text{mm}$$

BH evolution

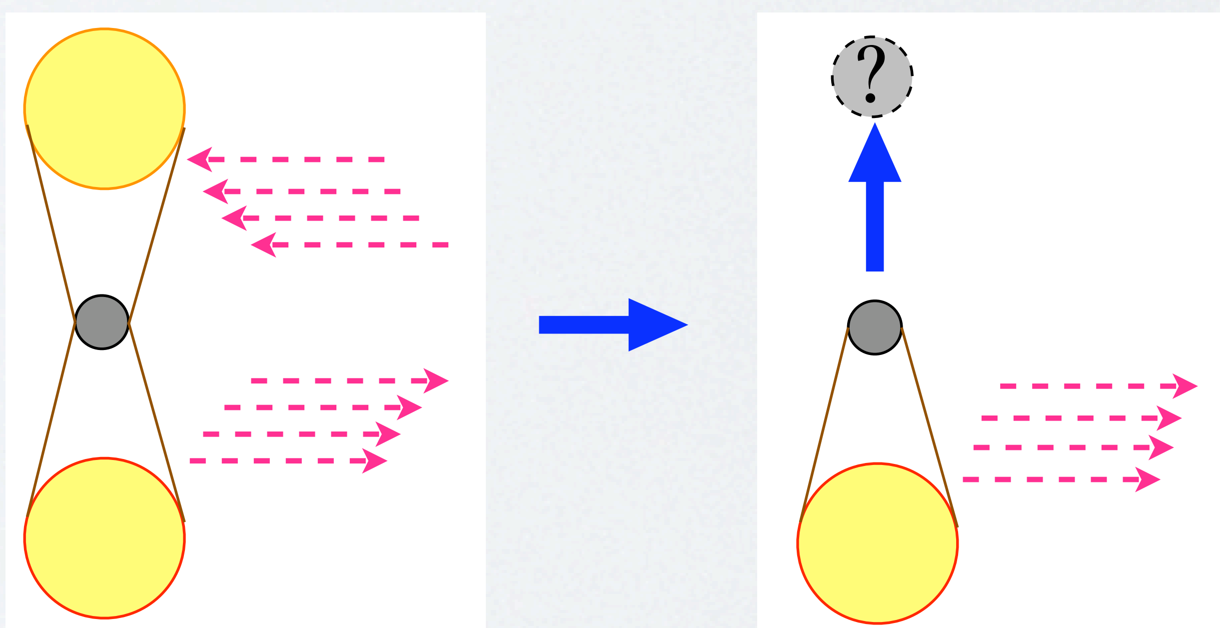
Trace "anomaly":

$$\mathcal{R} \equiv R^{\mu}{}_{\mu} + 8\,\pi\,T^{\mu}{}_{\mu} = -8\,\pi\,\frac{\rho^2}{\lambda} = -\frac{1}{2\,\pi\,\lambda}\left(\frac{M_0'}{R^2\,R'}\right)^2$$

At the OS "surface" $r = r_0$: $\mathcal{R} = -\dfrac{9}{2\,\pi\,\lambda}\,\dfrac{M_S^2}{R^6}$ (Hawking)

For $r > r_0$: $\mathcal{R} = O(\epsilon^2)$

Dissipation:



Minisuperspace WDW for dust

Dust star metric:

$$ds^2 = R\left[-d\eta^2 + \frac{d\rho^2}{1-\epsilon\,\rho^2} + \rho^2\,\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)\right]$$

$\epsilon = 0, \pm 1$
$0 < \rho < \rho_s$

WDW equation ($\hbar = 1$):

$$\left[\hat{H}_G + \hat{H}_M\right]\,\Psi \equiv \frac{1}{2}\left[\ell_p^2\,\frac{\partial^2}{\partial R^2}\,\frac{1}{R} - \frac{\epsilon}{\ell_p^2}\,R - \frac{1}{R^3}\,\frac{\partial^2}{\partial \phi^2} + \frac{1}{\ell_\phi^2}\,\phi^2\,R^3\right]\Psi(R,\phi) = 0$$

BO reduction:

$$\Psi(R,\phi) = R\,\psi(R)\,\chi(\phi,R)$$

$$\langle\hat{O}\rangle \equiv \int d\phi\,\chi^*\,\hat{O}\,\chi$$

$$\psi\,R\,\hat{H}_{_M}\,\chi + \ell_p^2\left(\frac{\partial\psi}{\partial R}\right)\frac{\partial\chi}{\partial R} \simeq 0$$

Minisuperspace WDW for dust

Introducing the time:

$$\psi_c(R) = \frac{e^{+i\int^R P_c\, dR_c}}{\sqrt{-P_c}} \qquad P_c = -\frac{1}{\ell_p^2}\frac{\partial R_c}{\partial \eta_c} = -\frac{1}{\ell_p^2}\sqrt{2\,\ell_p^2\, R_c\, \langle \hat{H}_M \rangle - \epsilon\, R_c^2}$$

$$\frac{\partial}{\partial \eta_c} \equiv -\ell_p^2\, \psi_c\, P\, \frac{\partial}{\partial R} = -\ell_p^2\, P_c\, \left.\frac{\partial}{\partial R}\right|_{R_c}$$

Semiclassical EHJ:

$$\frac{1}{2\,\ell_p^2}\left(\frac{dR_c}{d\eta_c}\right)^2 + \frac{\epsilon}{2\,\ell_p^2}\, R_c^2 - R_c\, \langle \hat{H}_M \rangle \simeq 0$$

Allowing for quantum fluctuations: $\psi = \psi_c\, f$

$$i\,\frac{\partial f}{\partial \eta_c} = \frac{1}{2}\left[\left(\frac{5}{4}\,\frac{\dot{P}_c^2}{P_c^4} - \frac{1}{2}\,\frac{\ddot{P}_c}{P_c^3}\right) f(R_c) + \frac{\dot{P}_c}{P_c^2}\,\left.\frac{\partial f}{\partial R}\right|_{R_c} + \ell_p^2\,\left.\frac{\partial^2 f}{\partial R^2}\right|_{R_c}\right]$$

Minisuperspace WDW for dust

Adiabatic limit: $\ldots \ll |\ddot{R}_c| \ll |\dot{R}_c| \ll R_c$

$$i\,\frac{\partial f}{\partial \eta_c} = \frac{\ell_p^2}{2}\,\frac{\partial^2 f}{\partial R^2}\Big|_{R_c} \quad \Longrightarrow \quad f_\ell = \exp\left\{ i\,\frac{\ell_p^2\,\eta_c}{2\,\ell^2} + i\,\frac{R_c}{\ell} \right\}$$

$$E_\ell = \boxed{-\frac{1}{2\,R_c}}\,\frac{\ell_p^2}{\ell^2} \qquad (d\tau = R_c\,d\eta_c)$$

N.B.: exponential waves $\ell = i\,l$ correspond to tunneling and are not considered...

"Improved" semiclassical limit:

$$\psi_\ell = \frac{e^{i\int^R P_\ell\,dR_\ell}}{\sqrt{-P_\ell}}\,e^{i\frac{R}{\ell}} \equiv \bar{\psi}_\ell\,\bar{f}_\ell$$

$$\frac{1}{2}\left[\left(\frac{dR_\ell}{d\eta_\ell}\right)^2 + \epsilon\,R_\ell^2 + \frac{\ell_p^4}{\ell^2}\right] = \ell_p^2\,R_\ell\,\langle \hat{H}_M \rangle$$

$\uparrow$ $R_\ell \to 0$

$$\Longrightarrow \quad R_\ell > R_{\min} \sim \left(\frac{E_\ell}{\langle \hat{H}_M \rangle}\right)^2 R_0$$

Core Bounce!

Conclusions

1. BW (holographic) improved OS gravitational collapse:
   i. stars have an "atmosphere"
   ii. "effective" pressure (mimics quantum mechanics)
2. Apparent horizon forms with Hawking-like radiation
3. No real singularity forms (similar to quantum mechanics)
4. System evaporates collapsing and then back...