INFN Istituto Nazionale di Fisica Nucleare

Dipartimento di Fisica "E.R. Caianiello"

Large N Gravity

Fabrizio Canfora

Classical and Quantum Gravity Group

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* Dipartimento di Matematica e Informatica
** Dipartimento di Fisica E.R. Caianiello

http://www.sa.infn.it/gaetano.vilasi/slides/

Motivations

- In QCD, the ‘t Hooft and Veneziano limits are one of the main tools to investigate non perturbative phenomena which standard perturbation theory is not able to disclose (such as confinement, baryons and mesons physics, chiral symmetry breaking and so on).
- Many models are not renormalizable in the standard perturbative expansion and, in fact, are renormalizable in the large N expansion.
- Thus, some sort of large N limit(s) would be very useful in General Relativity which is not perturbatively renormalizable and whose quantistic features are still far from being fully understood.

But…

- Unfortunately, at a first glance the large N expansion cannot be applied to General Relativity.
- In Gauge Theories the main fields are 1-forms taking values in the algebra of the gauge group: the large N limit is, basically, the limit in which the gauge group is larger and larger.
- In Gravity the main field is the space-time metric, a rank-two covariant tensor field: there is not a clear separation between space-time and internal symmetries because General Relativity is the theory of space-time itself.
- Nevetheless, it is possible to formulate General Relativity in a way which is very close to a Gauge Theory. This will help in formulating the large N limit as well as in comparing the Gravitational and Yang-Mills cases.

Review of large N QCD.

The Lagrangian and the basic fields are:

$$L = -\frac{1}{2e^2} trF_{\mu\nu}F^{\mu\nu} + \left(\overline{\psi}\right)^a \gamma^\mu \left(D_\mu\right)_{ab}\psi^b$$

$$\left(D_\mu\right)_{ab} = \partial_\mu \delta_{ab} - e\left(A_\mu\right)_{ab},$$

$$\left(F_{\mu\nu}\right)_{ab} = \left[D_{\mu a}, D_{\nu b}\right]$$

Propagators and Vertices

$$\left\langle A^{ab}{}_{\mu} A^{cd}{}_{\nu} \right\rangle(k) = \frac{1}{2}\left(\delta^{ad}\delta^{cb} - \frac{\delta^{ab}\delta^{cd}}{N_c} \right) P_{\mu\nu}(k)$$

$$P_{\mu\nu}(k) \approx -\frac{\delta_{\mu\nu}}{k^2 - i\varepsilon}$$

$$\left\langle \psi_{(j)}{}^{a} \psi_{(j)}{}^{b} \right\rangle(k) \approx -\frac{\delta_{\mu\nu}}{m_{(j)} + i\gamma^{\mu}k_{\mu} - i\varepsilon}\delta^{ab}$$

$$V^{(3)} \approx ie\left(\delta_{\rho\nu}(k-q)_{\mu} + \delta_{\rho\mu}(p-k)_{\nu} + \delta_{\mu\nu}(q-p)_{\rho} \right)$$

$$V^{(4)} \approx e^2\left(2\delta_{\rho\mu}\delta_{\beta\nu} - \delta_{\rho\beta}\delta_{\mu\nu} - \delta_{\beta\mu}\delta_{\rho\nu} \right)$$

$$V^{(M)} \approx -e\gamma_{\mu}$$

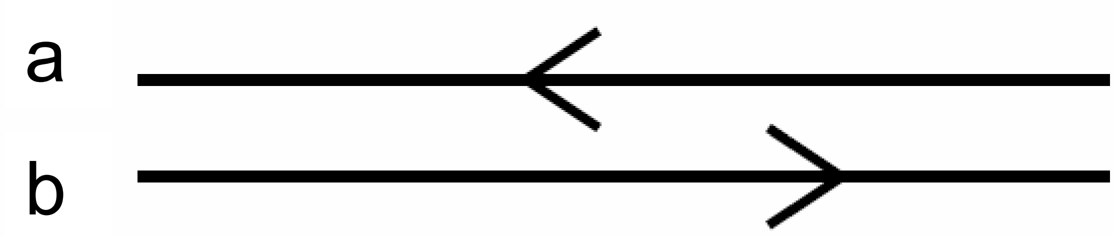
‘t Hooft notation

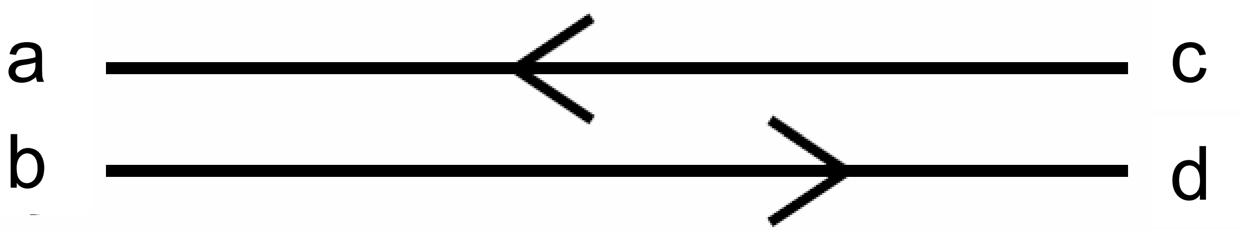
- A very clever way to take into account the internal index structures of the fields disentangling it from the space-time-momentum dependence in the path integral has been introduced by ‘t Hooft.

- To each gauge boson one has to associate two internal lines carrying internal indices with arrows pointing in opposite direction (in order to distinguish the fundamental and anti-fundamental representation of the gauge group SU(N)).

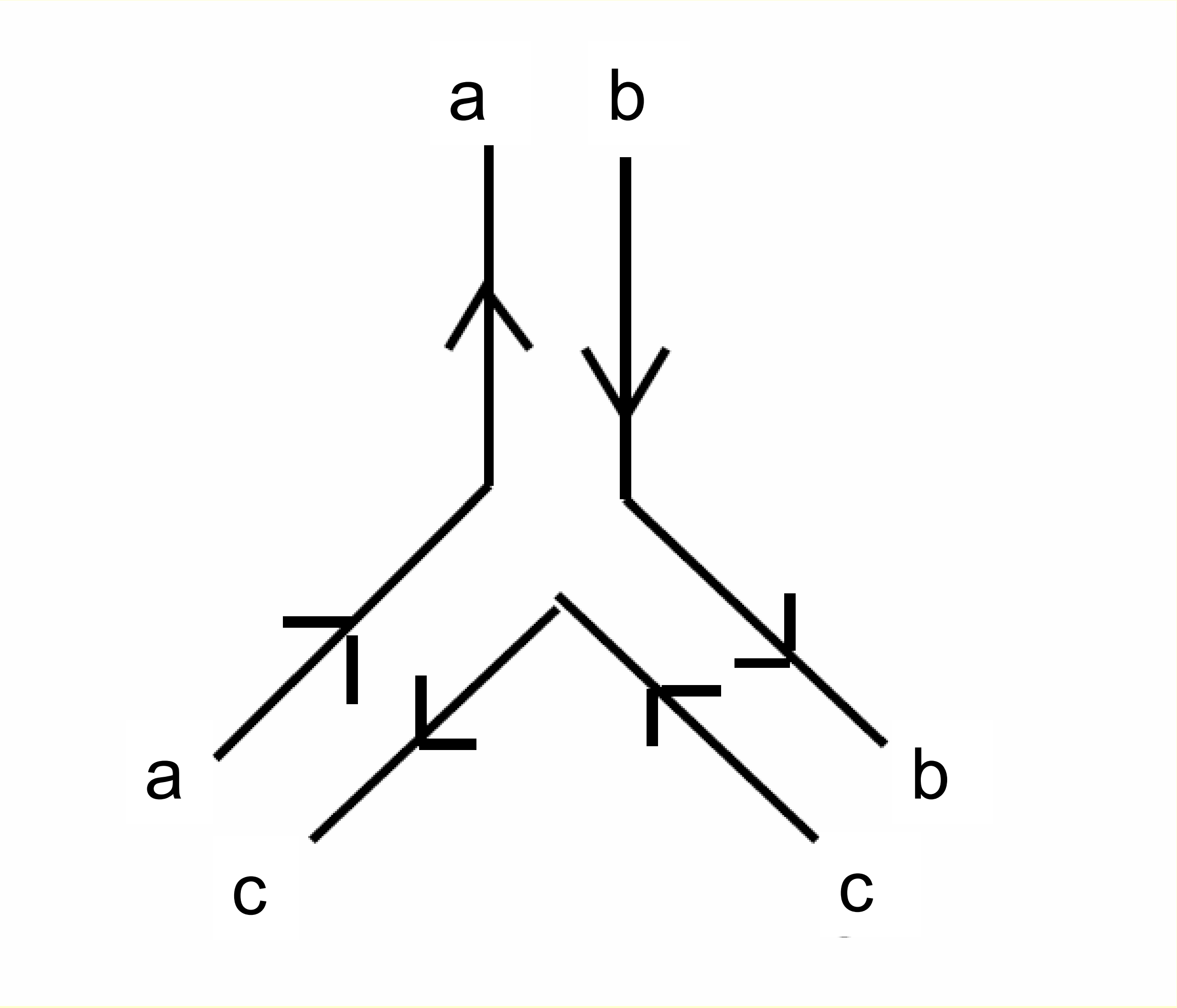
- To each quark one has to associate an internal lines carrying an internal index and an arrow, to each anti-quark one has to associate an internal lines carrying an internal index and an arrow pointing in the direction opposite to the quark arrow.

$\mathbf{A}_{ab}$

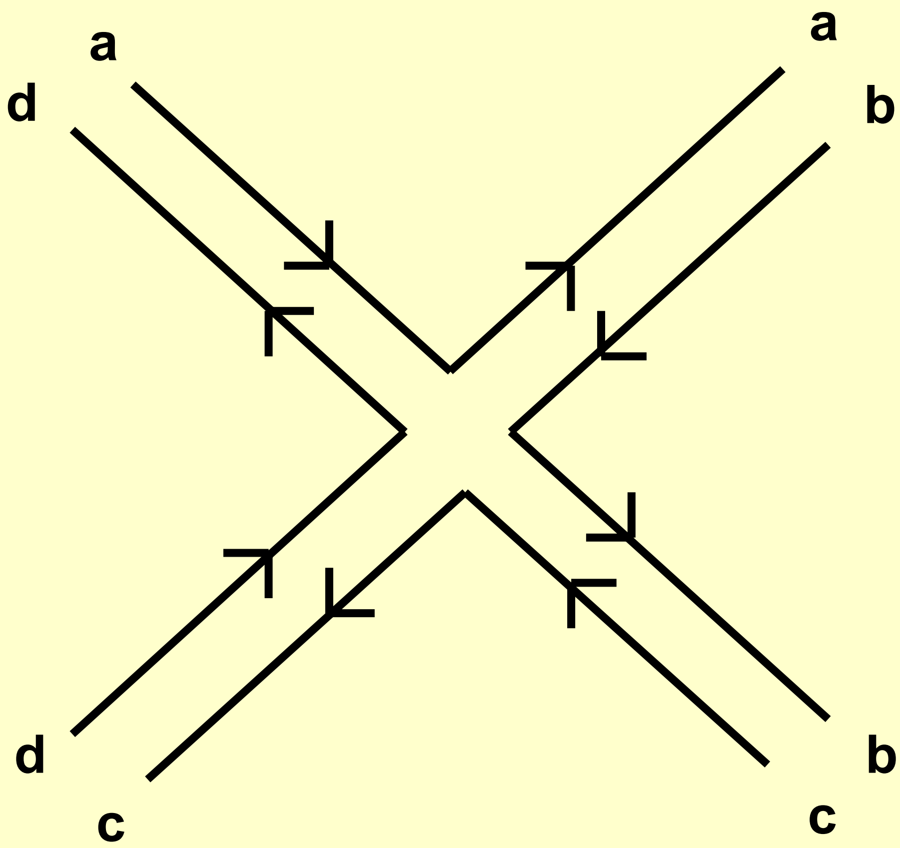




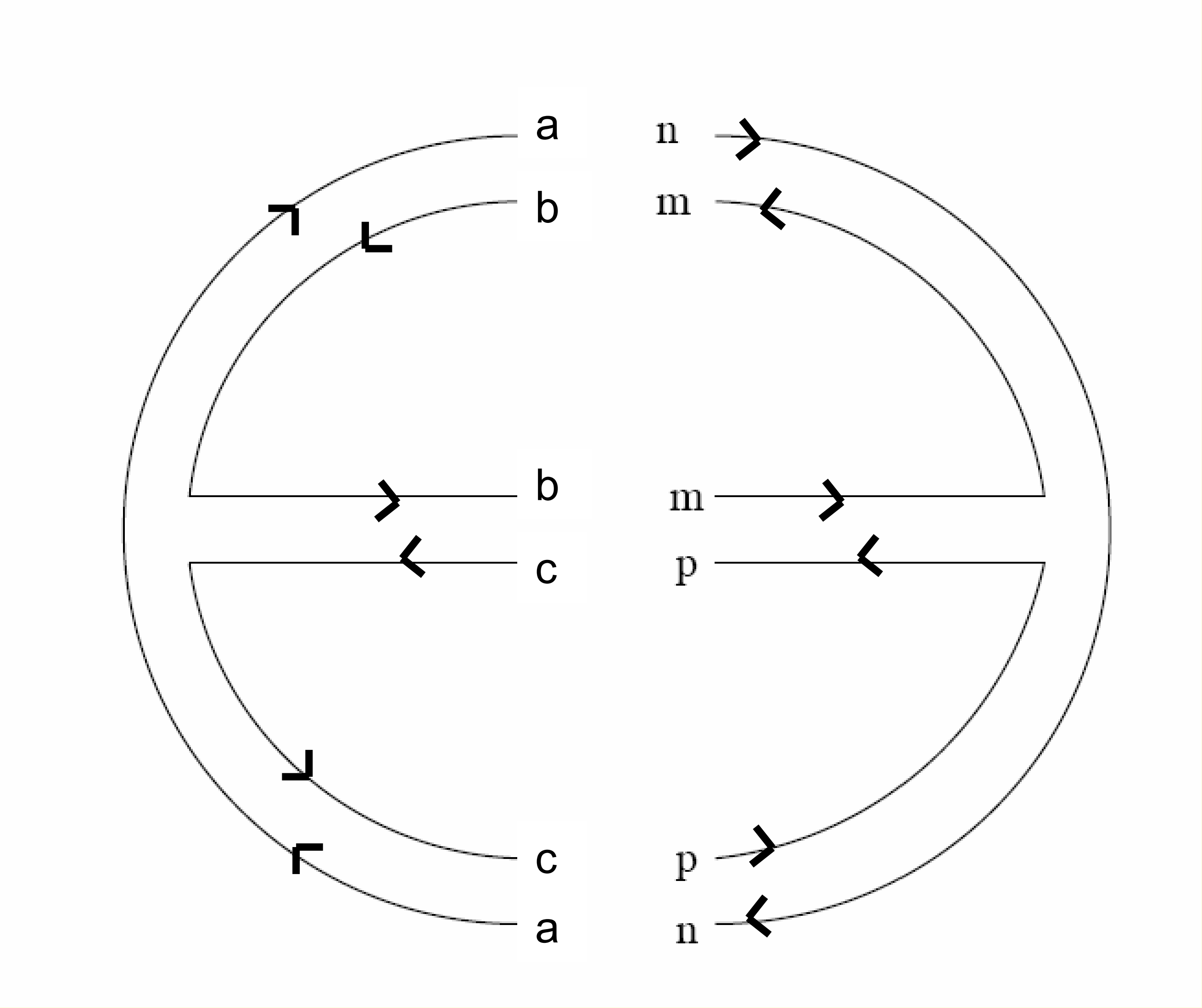
$\delta_{ac}\delta_{db}$



~e

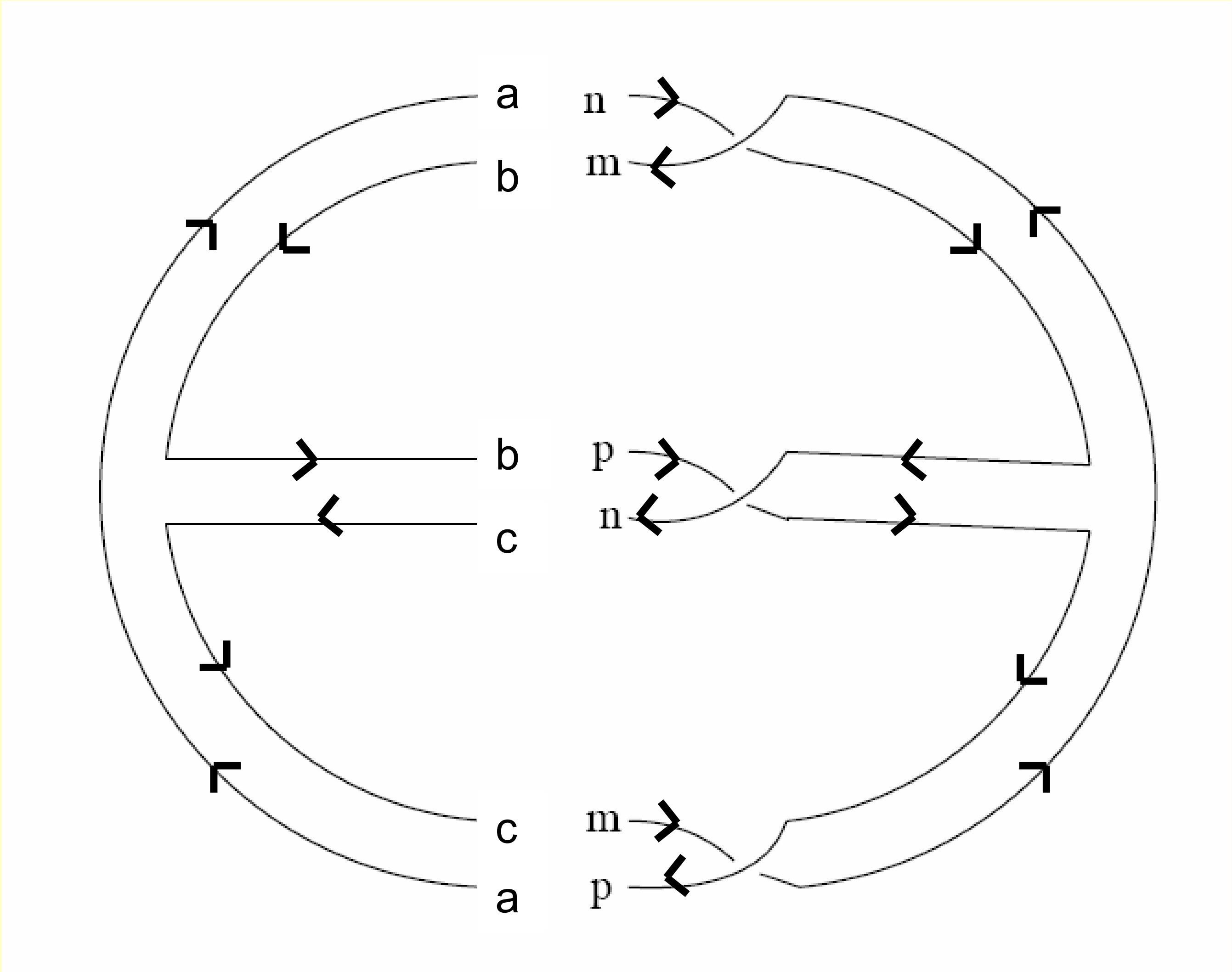


$\sim e^2$



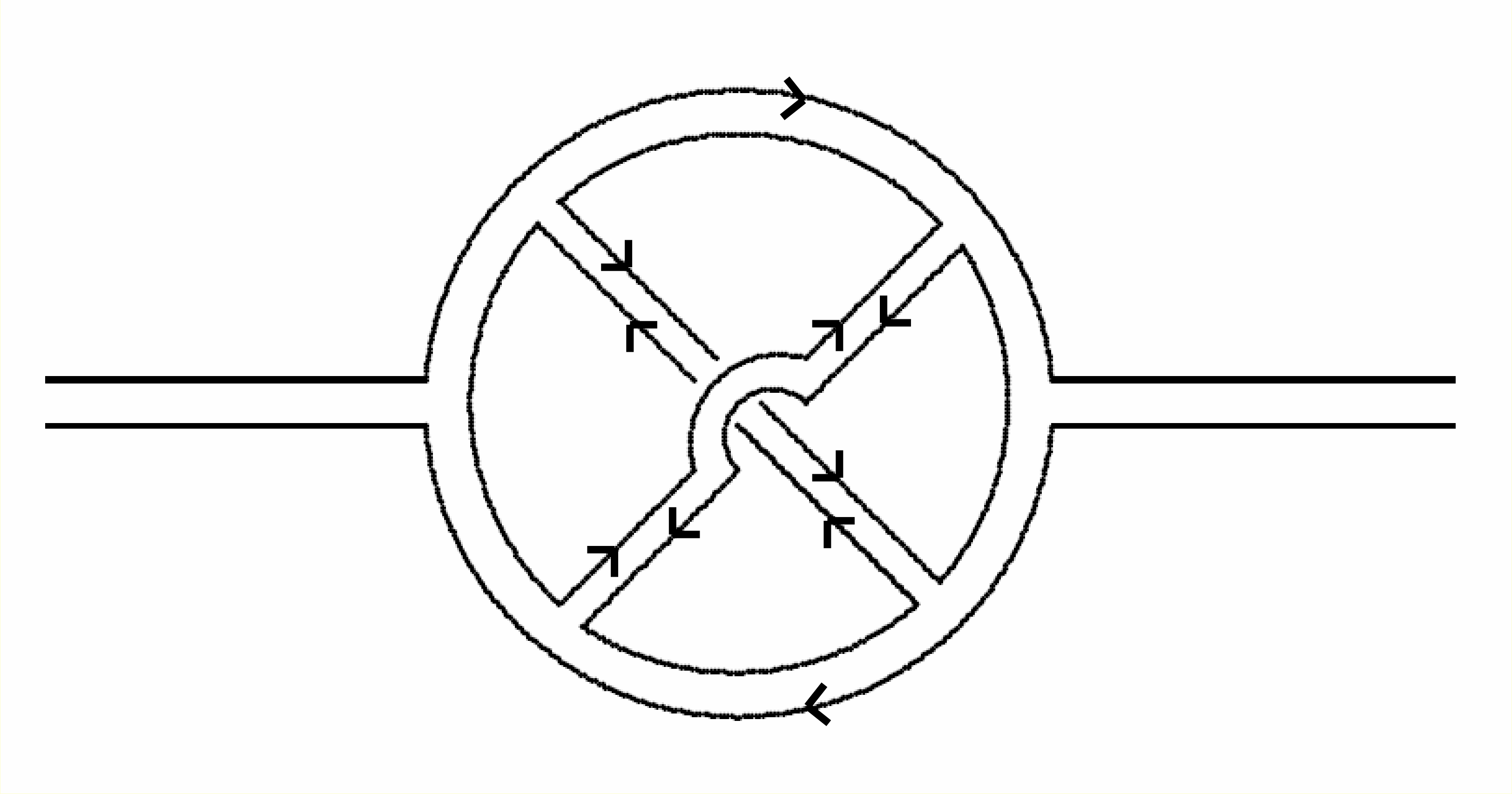
$\sim N^3$

$\sim e^2$

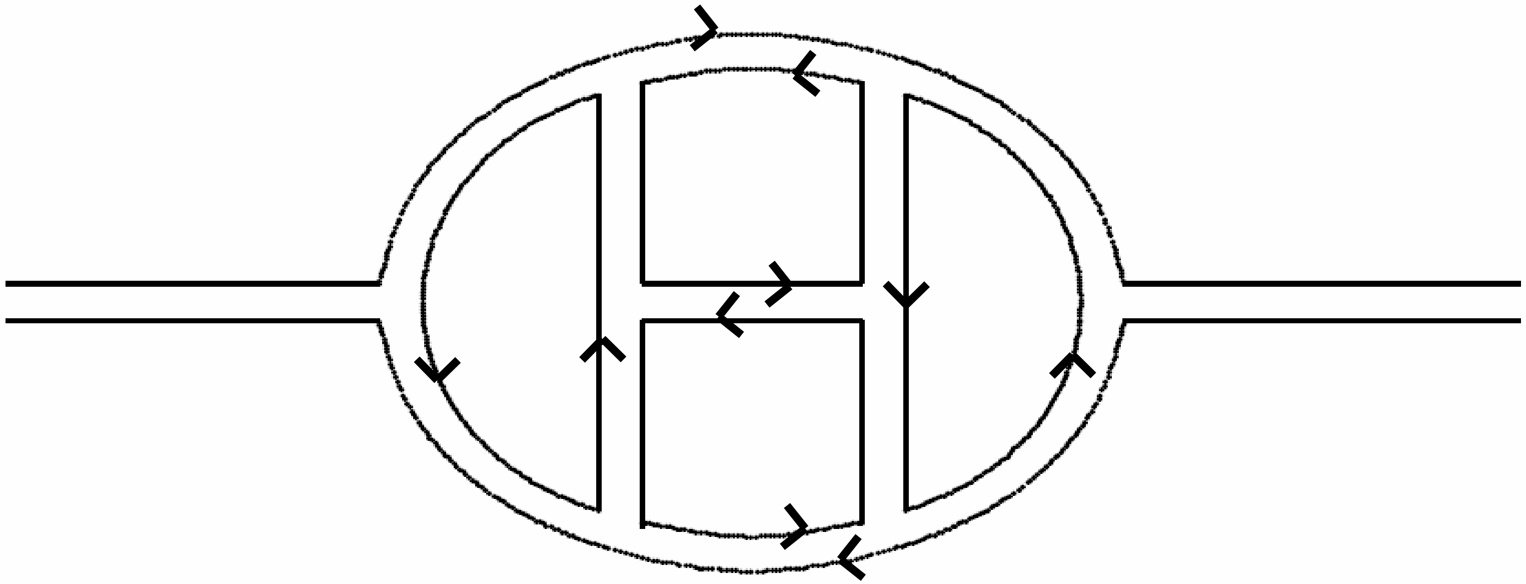


$\sim N$

$\sim \mathbf{e}^2$



Double-line representation of a three-loop non-planar diagram for the gluon propagator. The diagram has six three-gluon vertices but only one closed index line (while three loops!). The order of this diagram is e^6N.



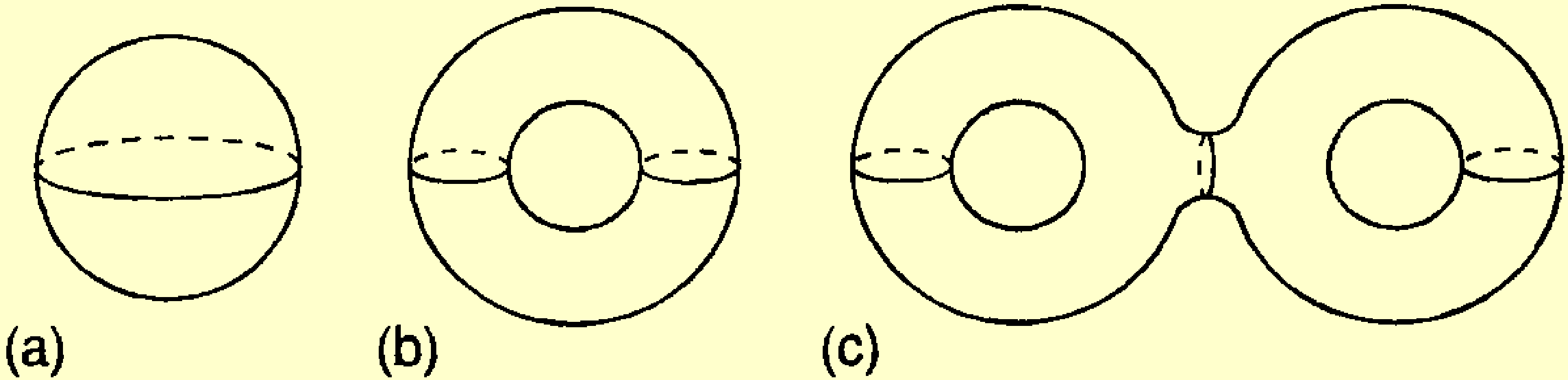
Double-line representation of a four-loop diagram for the gluon propagator. The sum over the Nc indices is associated with each of the four closed index lines whose number is equal to the number of loops. The contribution of this diagram is e^8N^4.

Large *N* Counting

$$W_{\Gamma}(k, x, g, L) \approx \lambda^{V_4+V_3/2}\left(N^{2-2g-L}\right)$$

$$\lambda = e^2 N$$

Thus, ‘t Hooft classified the large N diagrams according to their topological properties.
From the ‘t Hooft notation it is also clear that, in the topological expansion, only orientable surfaces enter since for SU(N) the fundamental representation is not real and the adjoint is the tensor product of the fundamental and the anti-fundamental representations. To derive this formula one only needs to use the Euler formula 2g-2=E-V-F.



Here one can see the first three orientable surfaces which enter in the topological expansion: in the case (a) the genus is zero and, therefore, the sphere gives the dominant contribution at large N, in (b) and (c) the genus is equal to one and two so that they are suppressed.

Summarizing the QCD case…

Thus, in gauge theories, the planar (genus zero) contribution is dominant. In the gluonic sector without quarks the non planar contributions are suppressed as $1/N^2$. The quark loops are suppressed as $1/N$. Confinement, baryons and mesons physics are well understood at large N.

BF formulation of Gravity

A very useful way to write the Einstein Hilbert action, well known in Loop Quantum Gravity, is as a topological action plus a constraint. It is available a seemingly similar formulation for gauge theories which allows interesting comparisons.

The action

$$\mathrm{S}_{\mathrm{EH}} = \frac{1}{G}\left[S(B,F) - \frac{c_2}{2}\int_M \phi_{abcd} B^{ab} \wedge B^{cd} + H(\phi) \right]$$

$$S(B,F) = \int_M B^{ab} \wedge F_{ab}(A)$$

$$\left(F_{\mu\nu}\right)_{ab} = \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)_{ab} + \left(A_\mu\right)^c{}_a \left(A_\nu\right)_{cb} - \left(A_\nu\right)^c{}_a \left(A_\mu\right)_{cb}$$

$$B^{ab} = \frac{1}{2} B^{ab}{}_{\mu\nu} dx^\mu \wedge dx^\nu$$

It is trivial to verify that, when one solves the equations for the non propagating field φ, the standard Palatini formulation of Einstein-Hilbert action is recovered. The BF theory is exactly solvable, thus gravity appears as an exact term plus a constraint. Yang-Mills theory also can be related to the BF action...

BF-Yang-Mills theory

$$\mathrm{S}_{\mathrm{YM}} = \frac{1}{G}\left[S(B,F) + e^2 \int_M B \wedge * B \right]$$

$$\left(F_{\mu\nu}\right)_{ab} = \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)_{ab} + \left(A_\mu\right)^c{}_a \left(A_\nu\right)_{cb} - \left(A_\nu\right)^c{}_a \left(A_\mu\right)_{cb}$$

$$B \wedge * B = tr B_{\mu\nu} B^{\mu\nu}$$

Thus, Yang-Mills action is a topological action plus a deformation: this formulation is useful to compare the two theories.

Propagators, Vertices,…

$$\left\langle A^{ab}{}_{\mu} B^{cd}{}_{\nu\rho} \right\rangle(k) = \frac{1}{2}\left(\delta^{ad}\delta^{cb}\right)P_{\mu\nu\rho}(k)$$

$$P_{\mu\nu\rho} \approx P_{\mu\nu\rho}(k)$$

$$V^{(3)}(A, A, B) \approx c_1$$

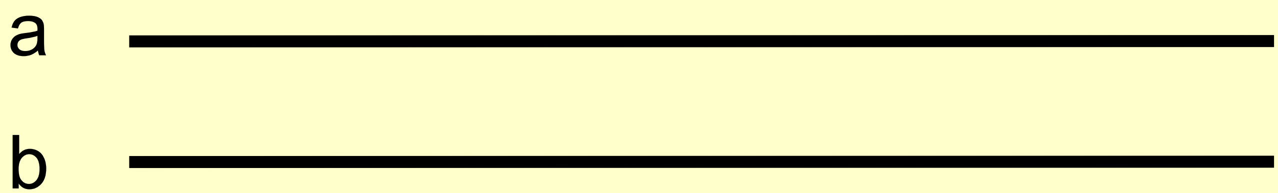
$$V^{(4)}(B, B, \phi) \approx \frac{c_2}{2}$$

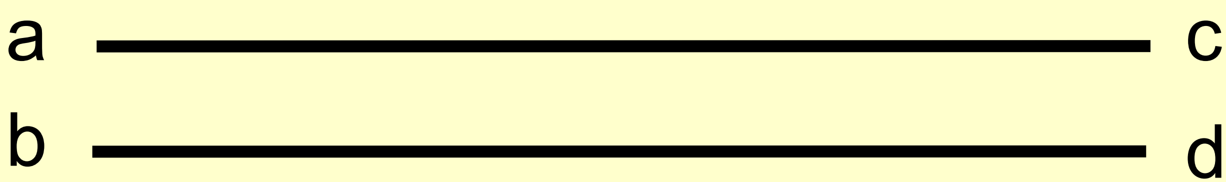
The first vertex is present both in the Yang-Mills case and in the Gravitational case; the second one only pertains to gravity. Also the propagators are different.

Gravitational ‘t Hooft notation

The ‘t Hooft notation can be introduced as before, being the physical fields in the adjoint of the gauge group. However, there is an interesting difference: being the fundamental representation of SO(**N**-1,1) (which, in the gravitational case, is the gauge group) real and the adjoint the tensor product of the fundamental by itself, there is no need to use the arrows. Physically, the “gravitational charge” is always positive.

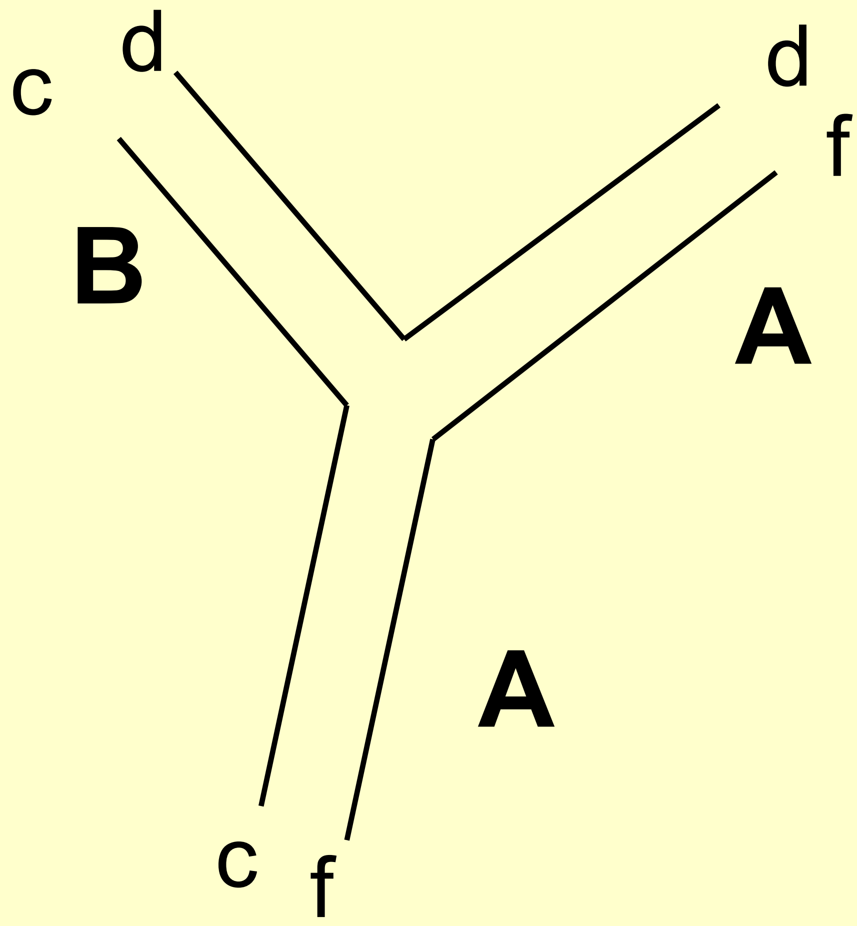
$A \to B$

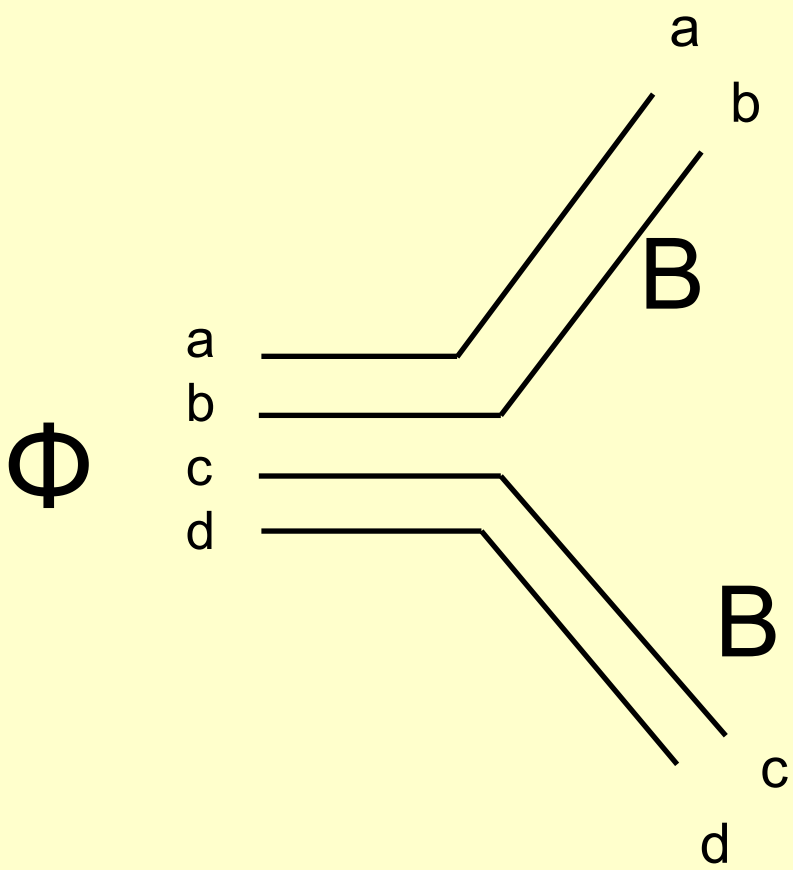




$$\delta_{ac}\delta_{db}$$

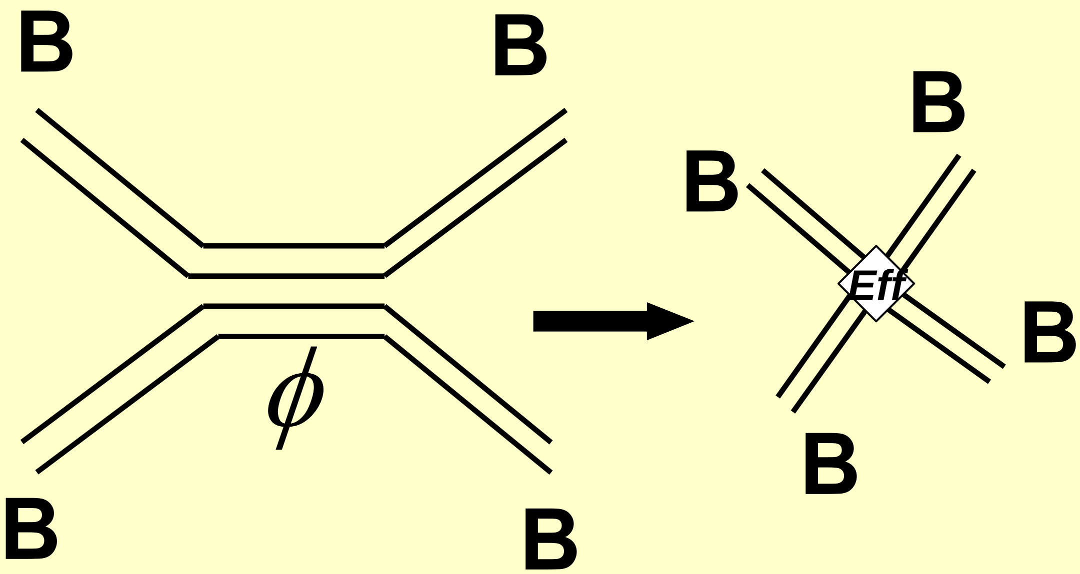
Vertices in the ‘t Hooft notation





This vertex gives rise to an effective fourth vertex (?).

A quadruple vertex?



In fact…

- At a first glance, it seems that the vertex with Φ gives rise to an effective quadruple vertex for **B**.
- In fact, as it can be seen graphycally, this is not completely true. This fact has an interesting physical interpretation.

Matter coupling

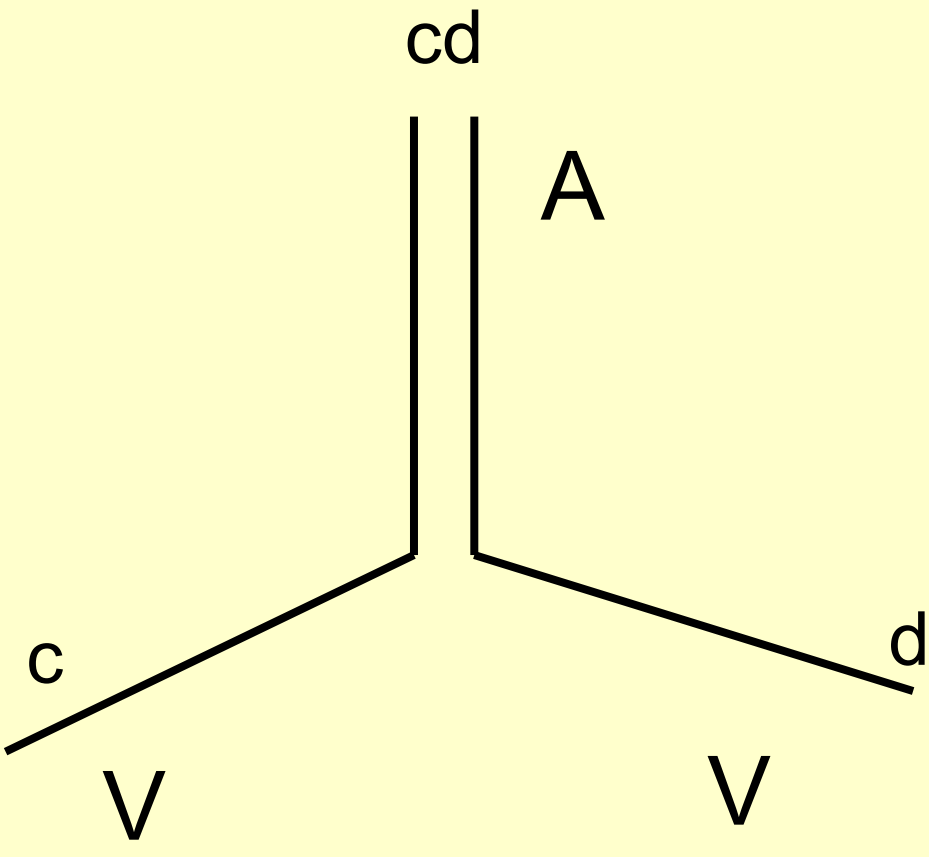
- Levi-Civita covariant derivative (in which it enters the gravitational connection) couples to tensorial indices.
- Thus, in this scheme, vector and spinor fields should be seen as scalar field with an internal index.

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu{}_{\mu\rho} V^\rho \Leftrightarrow$$

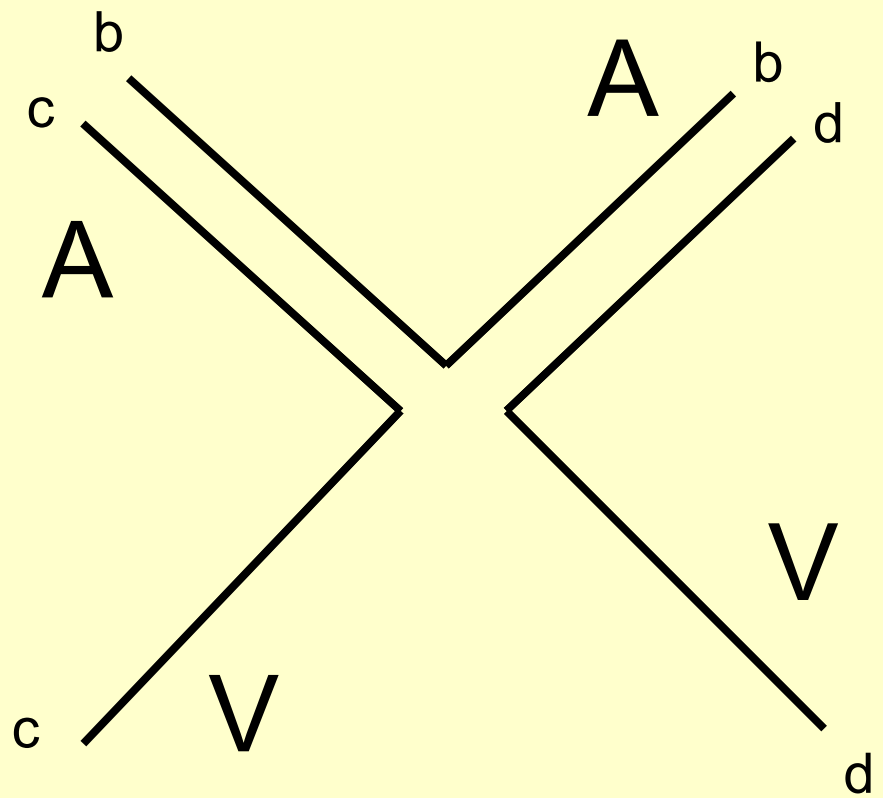
$$\left(\nabla_A V\right)^a = \partial_\mu V^a + \left(A_\mu\right)^a{}_b V^b$$

$$V^\mu \rightarrow \left(V\right)^a$$

Matter vertices



Thus, as in the Yang-Mills case, matter fields can be dealt as fields carrying an internal index: the counting will be also similar.



Large N expansion and gauge/gravity duality

- In many models inspired by string theory it happens that a strongly coupled gauge theory (on the boundary of suitable manifolds) “corresponds” to a weakly coupled supergravity theory in the bulk and viceversa.
- This correspondence would suggest that gravity should be strongly coupled in the UV in the same way as standard gauge theories are strongly coupled in the IR.
- The gravitational coupling with matter field tells that the Lorentz indices look like internal indices.

- Thus, one is lead to the possibility of a sort of gravitational confinement in the UV in which Lorentz indices are confined.
- It is also possible to provide an intuitive argument supporting this hypothesis.
- This phenomenon would greatly improve the UV behavior of gravity.

An intuitive argument…

$$\lambda_T \approx \frac{1}{T}$$

$$r_\lambda \approx GT \Rightarrow T \geq \frac{1}{\sqrt{G}} \Rightarrow \lambda_T \leq r_\lambda$$

However, one could add an extreme blackhole in such a way that the two heavy test particles are just outside the horizon on the opposite endpoints of a diameter. Since the extreme blackhole has a vanishing Hawking temperature it protects the test particles against the sea of mini blackholes emerging from the vacuum. Extreme blackholes in gravity play the same role as monopoles in QCD since they are BPS states. This argument would not work with non extreme blackholes.

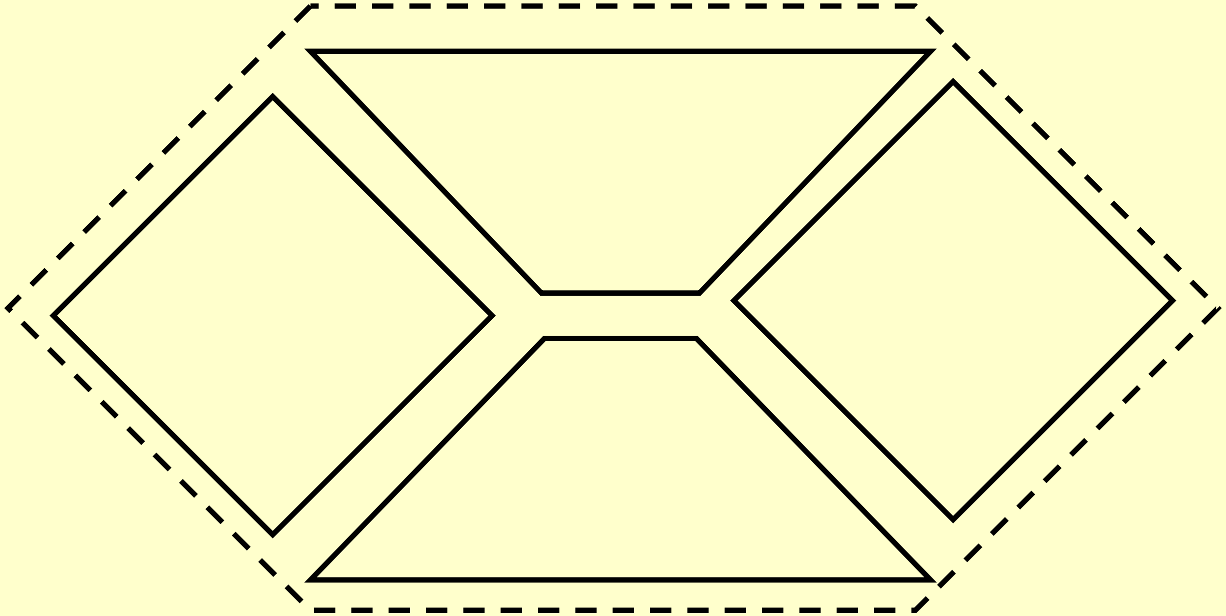
$$\Delta F \approx M = \sqrt{A} \approx L$$

Thus, the cost for maintaining the particles in the system is linear in the distance between the particles: confinement?
If this would be the case, there should be a great UV improvement:

$$A_V(s, t, J, M) \approx \frac{s^J}{t - M^2}$$

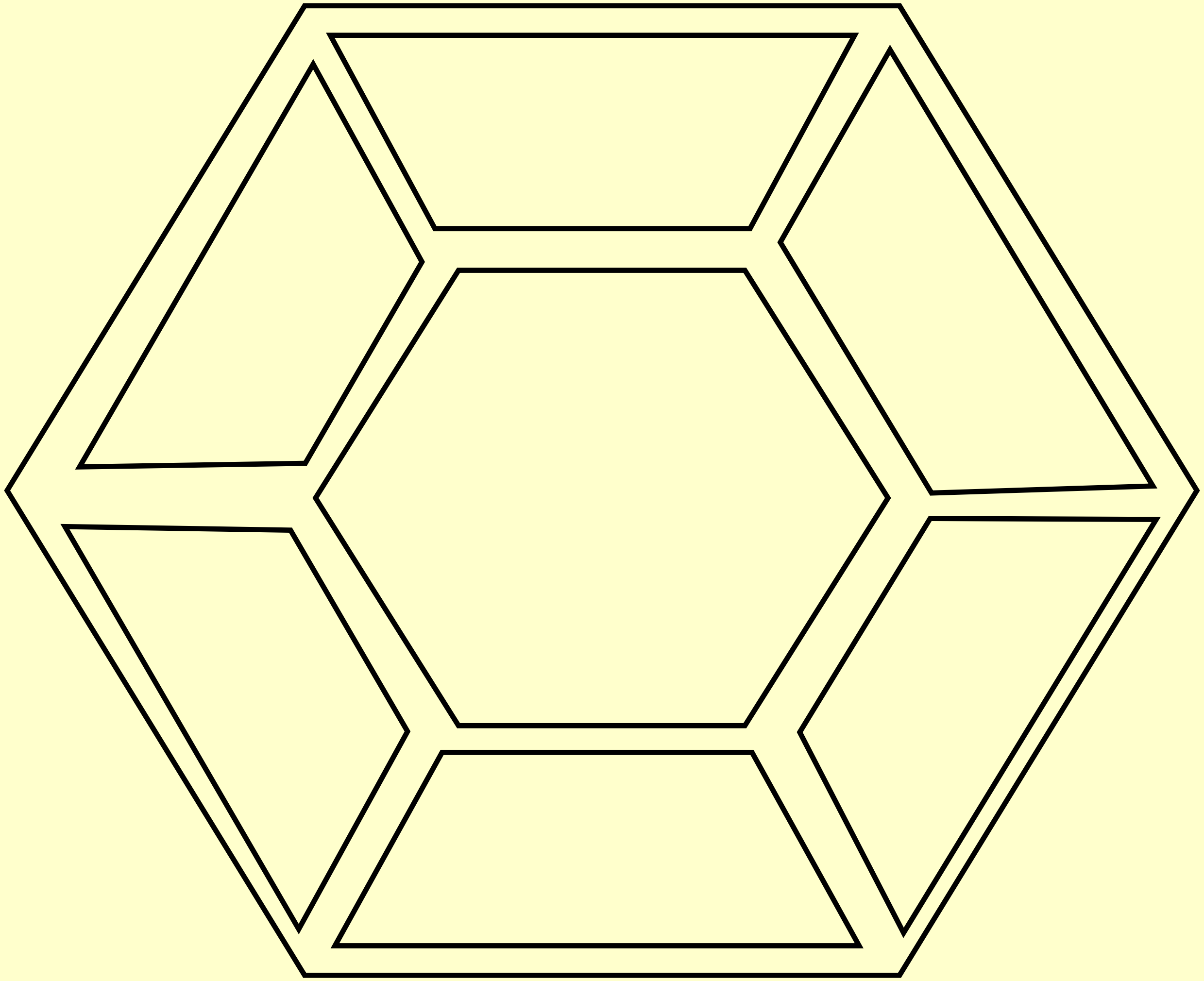
If, beyond a suitable energy scale, only scalar particles would be left in the spectrum, the above UV problem would be softened.

An example of a graph contributing to the free energy with one matter loop, four color loops, four matter vertices and two gravitational vertices.

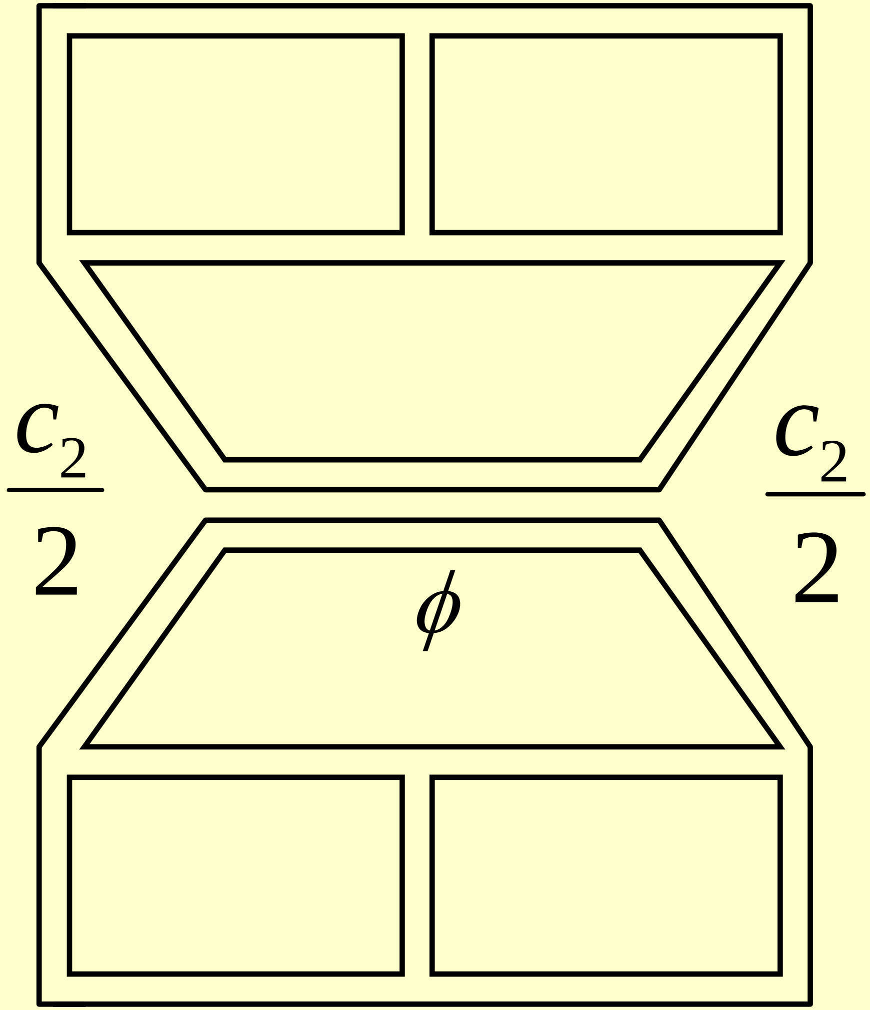


$$\approx N^4 N_f$$

An example of a graph contributing to the free energy with eight color loops.



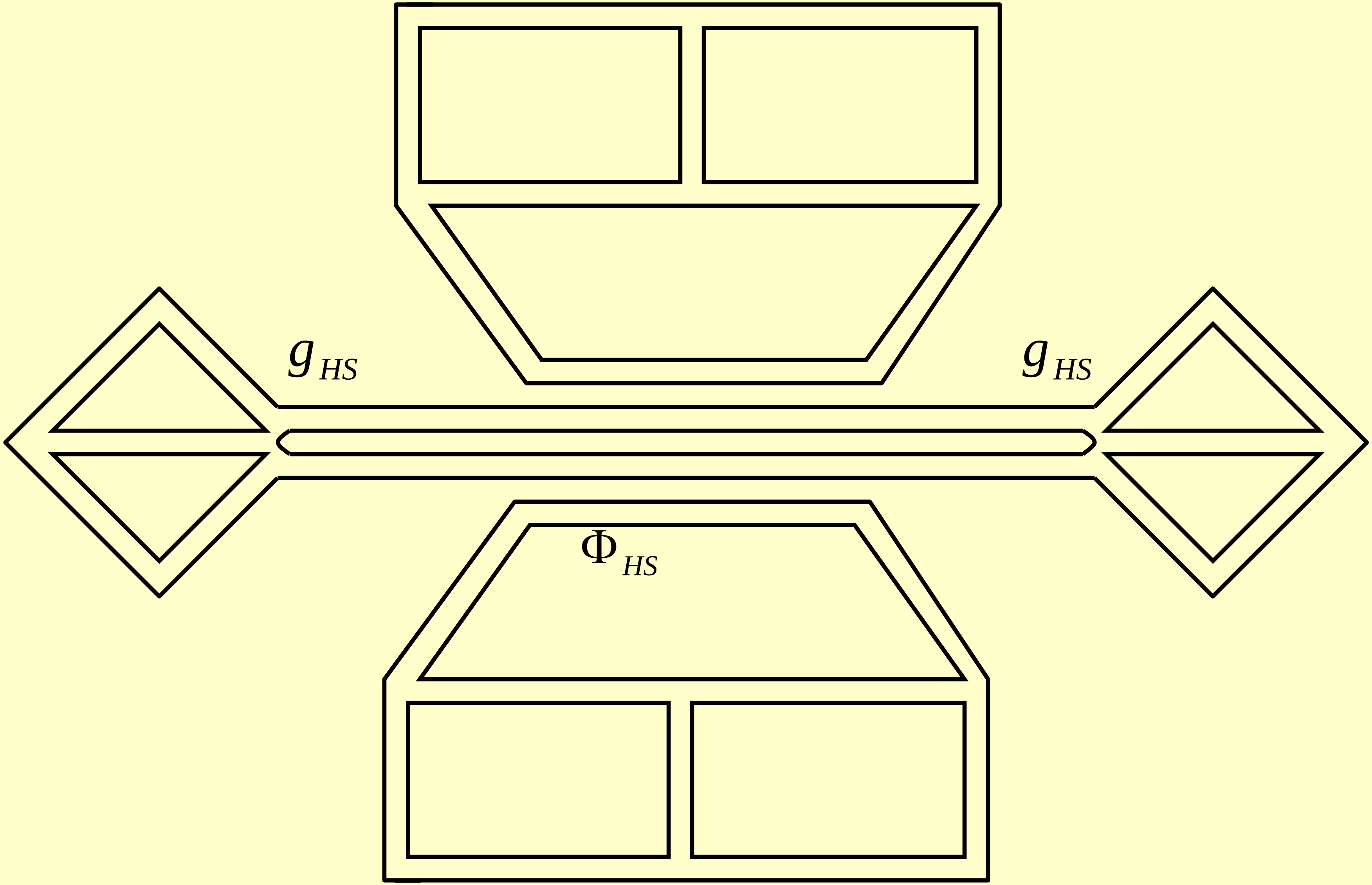
$$\approx N^8$$



$\frac{c_2}{2}$
$\frac{c_2}{2}$
ϕ

Here (one of) the difference(s)…

- The previous kind of graphs, which only pertain to gravity, would have no role in a gauge theory with fields carrying only one and two internal indices.
- In fact, in gravity, it naturally appears a field carrying four internal indices.
- This field is likely to have the role to decrease the entropy with repsect to the gauge theory case. This is the first quantistic argument supporting the holographic principle.



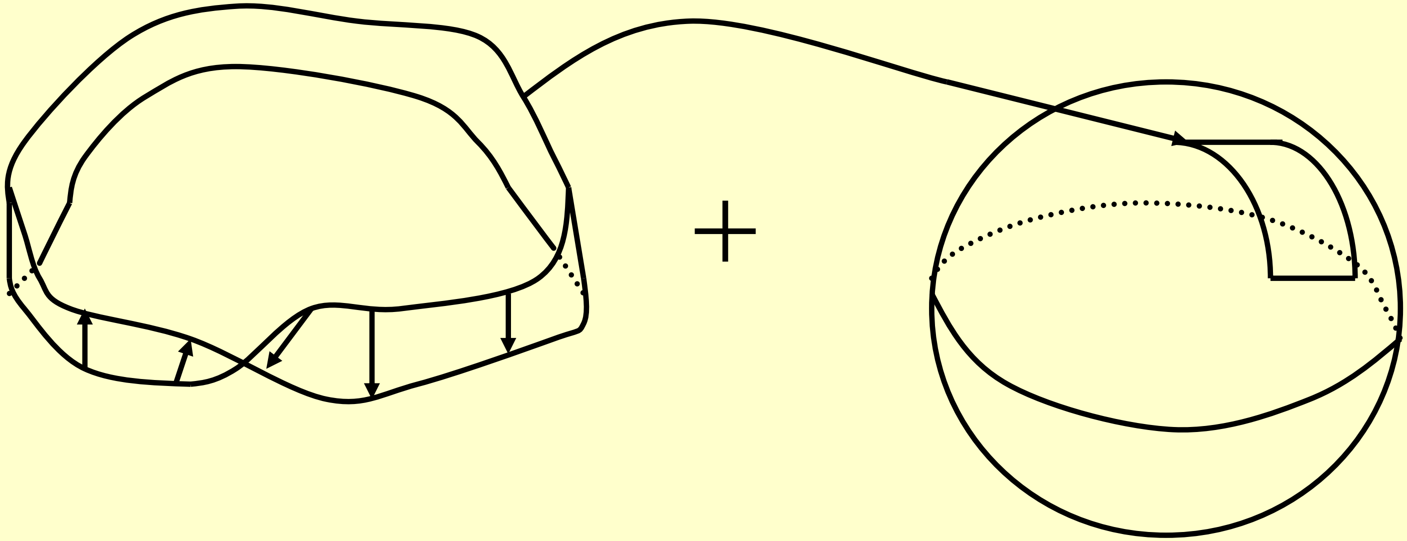
g_{HS}
g_{HS}
Φ_{HS}

Another difference…

- There is another difference related to the group-theoretical structure of gravity: internal lines carry no arrow. This implies that, in the topological expansion, non orientable surfaces cannot be omitted.
- Thus, purely gravitational loops, unlike gauge theory, are suppressed as 1/**N** instead of 1/$\mathbf{N}^2$.

The point is that the Euler formula *2g-2=E-V-F* also holds in this case provided the genus *g assumes half-integer values too.*

Gluing the boundaries



- The interpretation of this result is that, unlike gauge fields, the gravitational field is also able to imitate matter fields. In fact, this should not be too surprising: besides the Kaluza-Klein mechanism, exact solutions of vacuum Einstein equations carrying spin ½ and spin 1 have been found. This property could also be kept in the would be quantum theory.
- Eventually, it is interesting to note that, in some sense, scalar fields behave as baryons in QCD: this could explain (at least at a qualitative level) why they are so heavy and weakly interacting.

Conclusion

- The large N expansion in gravity seems to be a good tool to explore the strongly coupled regime of gravity. It is qualitative consistent with known results, it also provides the holographic principle with a field theoretical basis.
- Renormalization in 1/N, topology changes, fermions…

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